

Design a PDA which accepts:

$$L = \{ a^n b^m : n \neq m \}$$

Announcements

On Friday I would like to pick a time for a final exam tutorial.

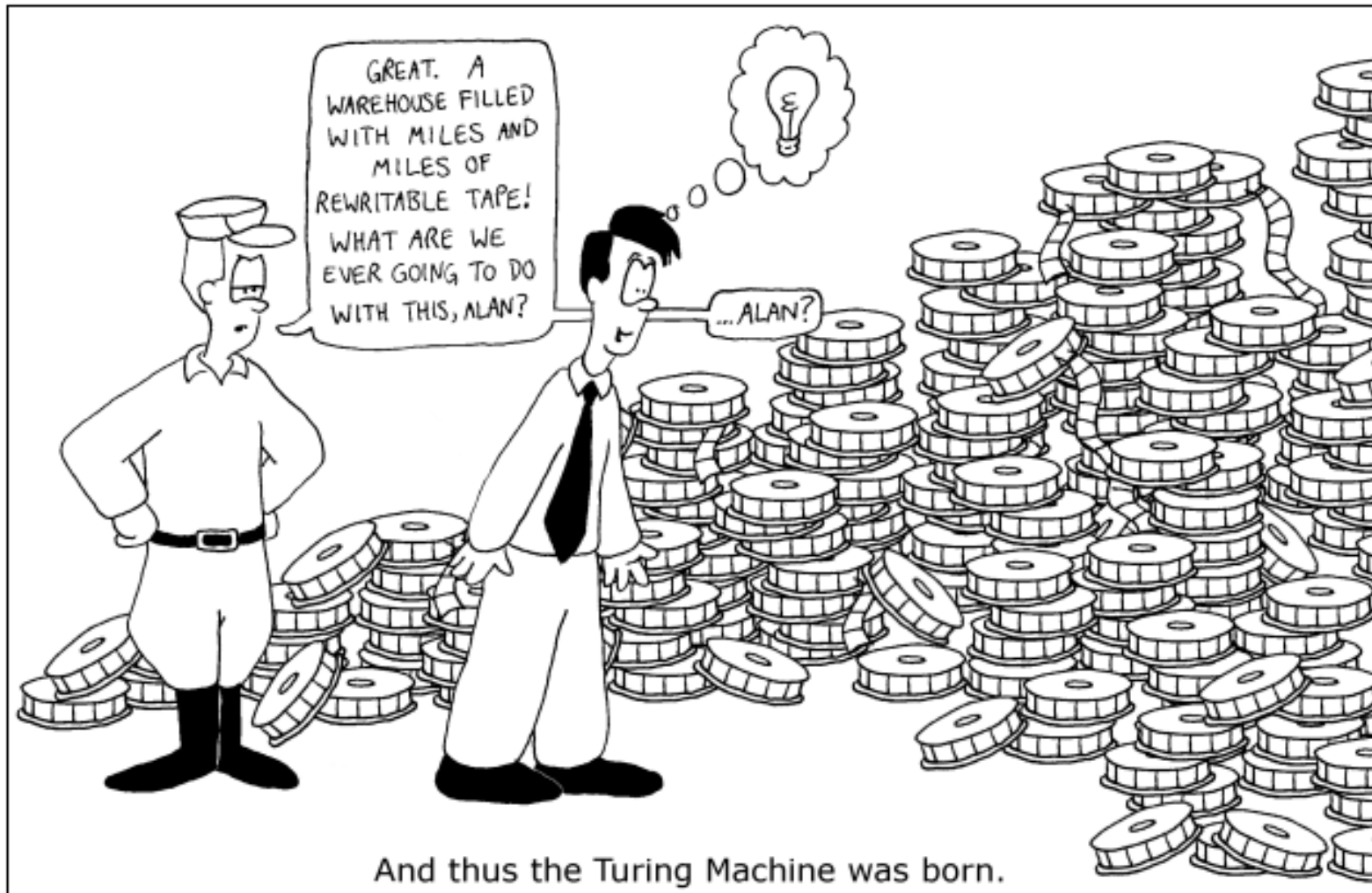
Our final exam is Tues. Dec. 6, 2pm.

Check your final exam schedule for conflicts on Monday Dec. 5.

Assignment #4 has been posted: due Friday Nov. 18.

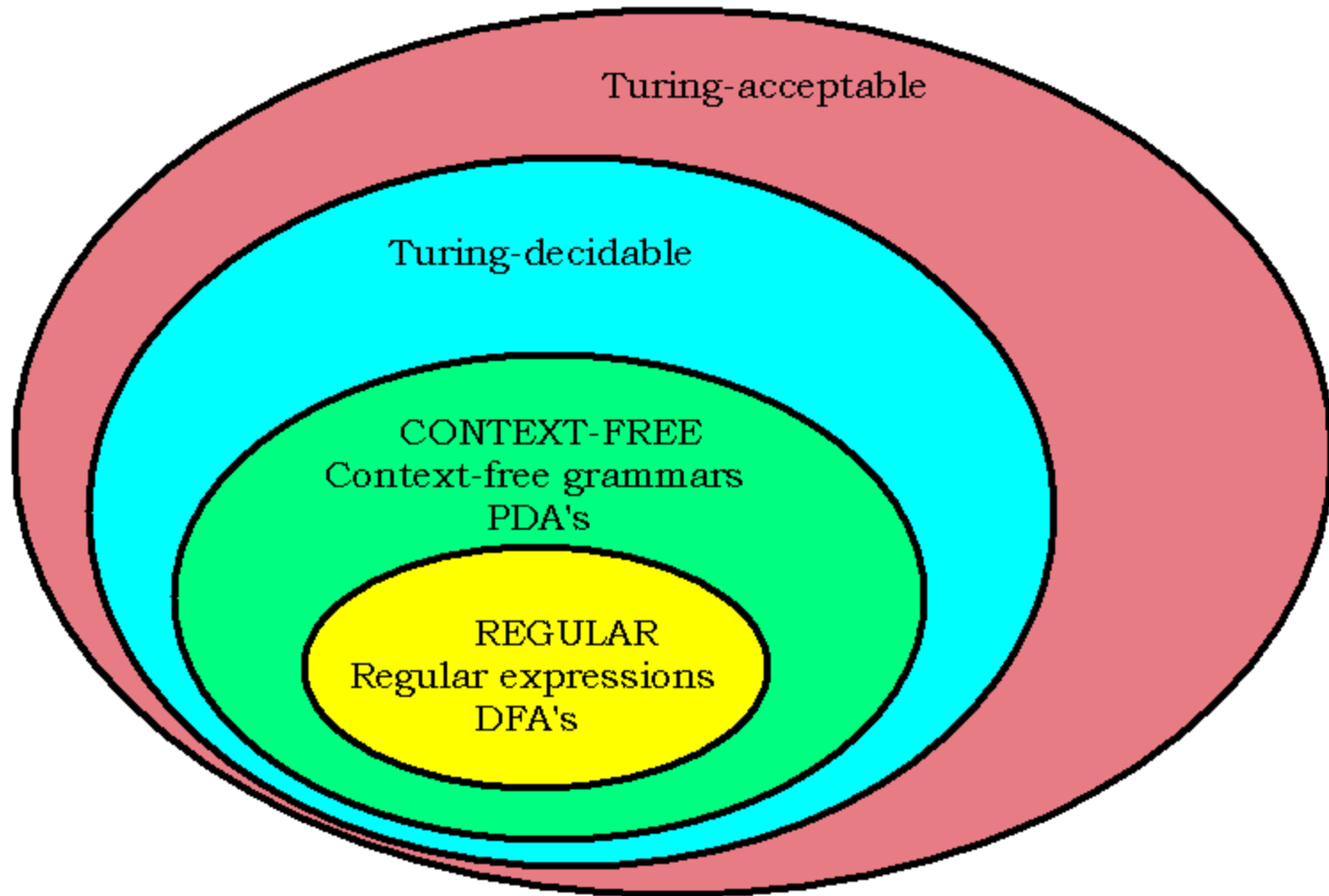
No tutorial this week, next week available but not mandatory.

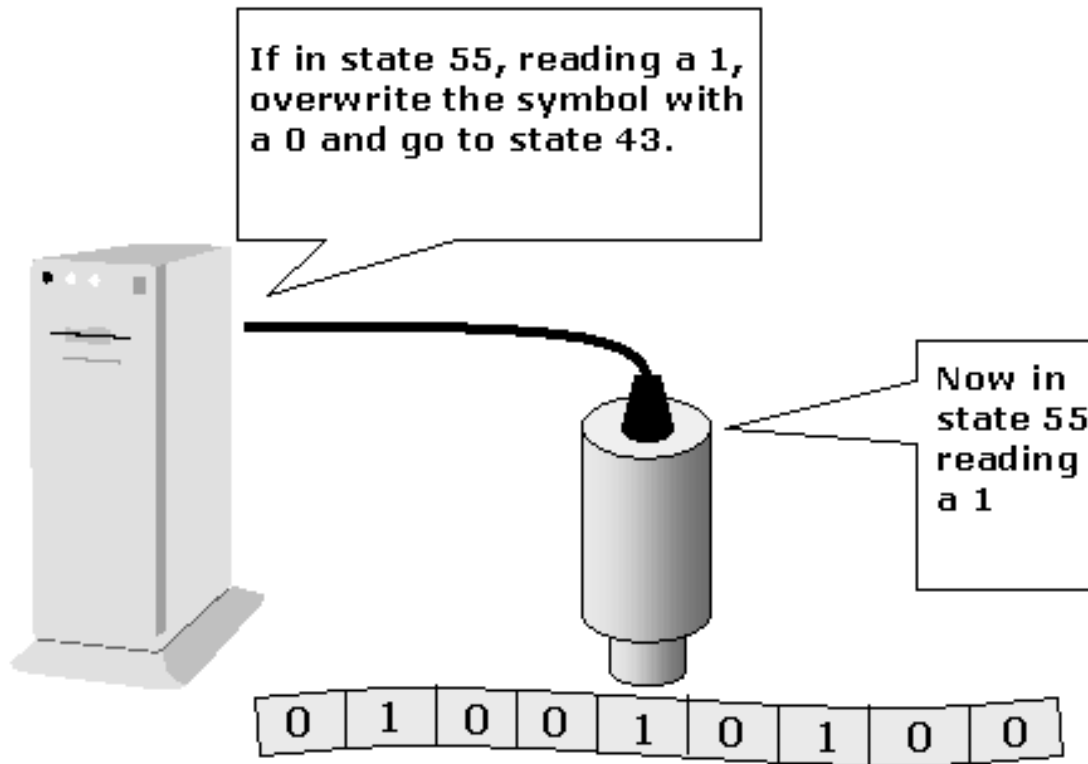
Turing Machines



This cartoon appears in the book *Computation Engineering: Applied Automata Theory and Logic* by Dr. Ganesh Gopalakrishnan.

Classes of Languages



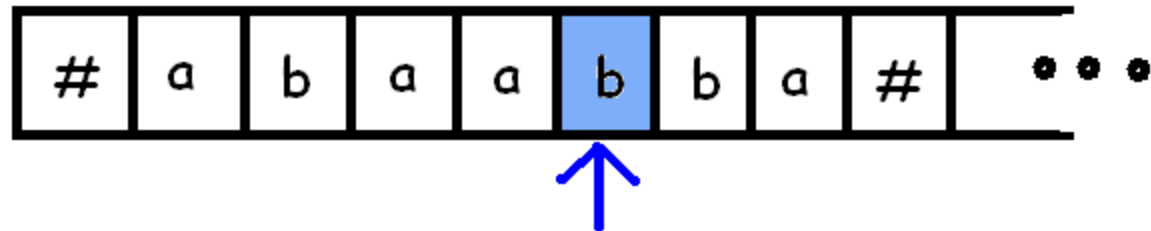


<http://www.lambdassociates.org/webbook/chap1.htm>

Turing machines are very simple (so it is easy to prove things about them) but are as capable as any current computer.

Turing machines:

Operation: add read/write tape to DFA.



Move:

Based on current state and symbol scanned:

1. change states, and
2. either replace tape square contents with a symbol or move head one square left or one square right.

Input conventions:

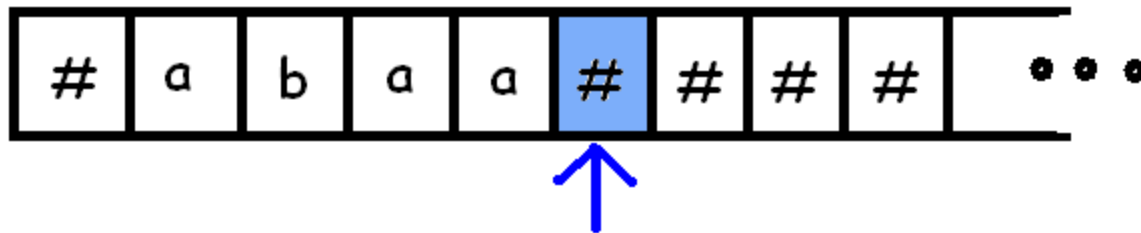
Tape has a left hand end but extends infinitely to the right (one-way infinite tape).

All squares are initially blank- represented by the symbol #.

On an input w , the tape starts out as:

w [#] (the [] contain the symbol scanned).

For example, on input $w = abaa$:



Special
halt state
called h .

$L = \{ u u^R : u \in \{a, b\}^* \}.$

Design a TM which **accepts** L :

It halts on input w if w in L and either hangs or computes forever when w is not in L .

A TM **halts** immediately when entering the special halt state h .

A TM **hangs** if the head falls off the left hand end of the tape or if it encounters an undefined transition.

Pseudo code for my algorithm:

1. Find leftmost input symbol and erase it.
If there is no input left at this stage, halt.
2. Move head to right hand end of input using the state to remember the symbol at the left hand end of the tape.
3. If the symbol does not match, make the TM hang. Otherwise erase the symbol and go back to step 1.

TM rules: State Symbol Next state Head instruction

// Move left to leftmost symbol.

Rule 1: start # left L

Rule 2: left a left L

Rule 3: left b left L

// When we find # at LH end, move R to check LH symbol.

Rule 4: left # check R

// If blank we are done- halt.

Rule 5: check # h #

// Blank out LH symbol and remember it using state.

Rule 6: check a rema #

Rule 7: check b remb #

// Move off blank remembering LH symbol using state.

Rule 8: rema # righta R

Rule 9: remb # rightb R

// Move to RH symbol remembering symbol using state.

Rule 10: righta a righta R

Rule 11: righta b righta R

Rule 12: rightb a rightb R

Rule 13: rightb b rightb R

// Check symbol at LH end.

Rule 14: righta # checka L

Rule 15: rightb # checkb L

// If it matches, blank it out and start over.

Rule 16: checka a start #

Rule 17: checkb b start #

// If not, hang because no transitions defined from undef.

Rule 18: checka b undef b

Rule 19: checka # undef #

Rule 20: checkb a undef b

Rule 21: checkb # undef #

An accepting computation:

(start, #abba[#])
- (left, #abb[a])
- (left, #ab[b]a)
- (left, #a[b]ba)
- (left, #[a]bba)
- (left, [#]abba)
- (check, #[a]bba)
- (rema, #[#]bba)
- (righta, ##[b]ba)
- (righta, ##b[b]a)
- (righta, ##bb[a])
- (righta, ##bba[#])
- (checka, ##bb[a])

- (start, ##bb[#])
- (left, ##b[b])
- (left, ##[b]b)
- (left, #[#]bb)
- (check, ##[b]b)
- (remb, ##[#]b)
- (rightb, ###[b])
- (rightb, ###b[#])
- (checkb, ###[b])
- (start, ###[#])
- (left, ##[#])
- (check, ###[#])
- (h, ###[#])

A non-accepting computation:

1. (start, #aaba[#])
2. (left, #aab[a])
3. (left, #aa[b]a)
4. (left, #a[a]ba)
5. (left, #[a]aba)
6. (left, [#]aaba)
7. (check, #[a]aba)
8. (rema, #[#]aba)
9. (righta, ##[a]ba)
10. (righta, ###a[b]a)
11. (righta, ##ab[a])
12. (righta, ##aba[#])
13. (checka, ##ab[a])
14. (start, ###ab[#])
15. (left, ###a[b])
16. (left, ##[a]b)
17. (left, #[#]ab)
18. (check, ##[a]b)
19. (rema, ##[#]b)
20. (righta, ####[b])
21. (righta, ####b[#])
22. (checka, ####[b])
23. (undef, ####[b])

No valid transition,
TM hangs.

Input to TM simulator:

// This TM halts if the input is of the form $u u^R$ where u
// is in $\{a,b\}^*$ and hangs otherwise.

start

// Move left to leftmost symbol.

start # left L // start algorithm to check if $u u^R$

left a left L // Go left to LH end of input

left b left L // Go left to LH end of input

// When we find # at LH end, move R to check LH symbol.

left # check R // Found # at LH end

// If blank we are done- halt.

check # h # // Done and answer is yes if input is all blank

// Blank out LH symbol and remember it using state.

check a rema # // Blank out LH symbol remember "a" using state

check b remb # // Blank out LH symbol remember "b" using state

// Move off blank remembering LH symbol using state.

rema # righta R // Start going right (remember "a")

remb # rightb R // Start going right (remember "b")

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// Move right to RH symbol remembering LH one using state.
righta a righta R // Move right (remember "a")
righta b righta R // Move right (remember "a")
rightb a rightb R // Move right (remember "b")
rightb b rightb R // Move right (remember "b")
// Check symbol at LH end.
righta # checka L // Found RH end- check now if "a"
rightb # checkb L // Found RH end- check now if "b"
// If it matches, blank it out and start over.
checka a start # // First symbol matches last one- start again
checkb b start # // First symbol matches last one- start again
// If not, hang because no transitions defined from undef
checka b undef b // No match- move to state with no transitions defined
checka # undef # // No match- move to state with no transitions defined
checkb a undef b // No match- move to state with no transitions defined
checkb # undef # // No match- move to state with no transitions defined
```

\$

```
abba // A string which is in L
aaba // A string which is not in L
```

The format of the input to the TM simulator is as follows:

<Name of start state>

<current state> <current symbol> <next state> <head instruction>

...

<current state> <current symbol> <next state> <head instruction>

\$

<input string w1>

<input string w2>

...

Rules for TM Descriptions

1. The state name `h` is used to denote the halting state.
2. Use the symbol `#` to represent a blank.
3. If a line starts with `//` it is a comment. Comments can also be added on the same line as an instruction at the end of the line (start with `//`).
4. Each of the state names is an arbitrary string. The current symbol and new symbol each must be a single symbol.
6. The head instruction is either `L` (move the head one square left) or `R` (move the head one square right) or a symbol to replace the current tape square contents.
7. The `$` indicates the end of the TM description.