1. Which of these sequences correspond to Hamilton cycles in the graph?

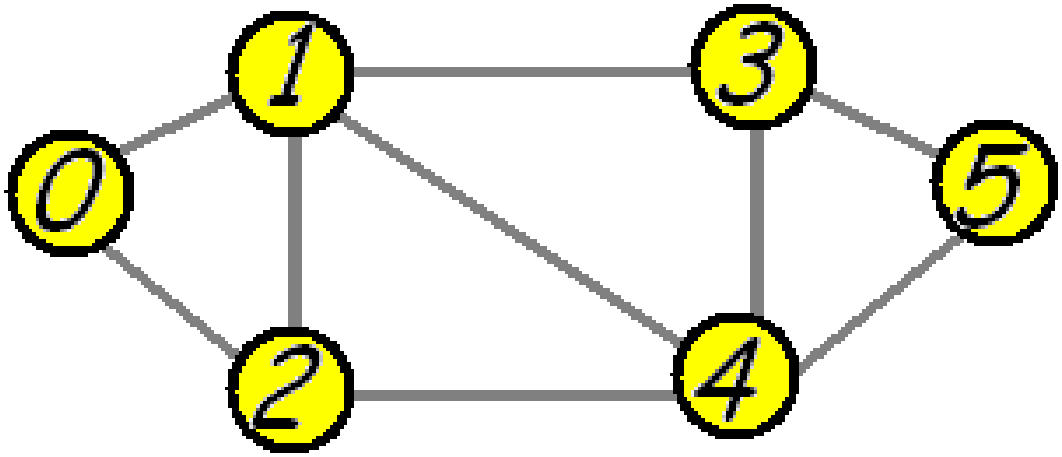
(a) 0 1 3 5 4 2

(b) 0 1 2 4 3 5

(c) 0 1 4 3 1 2

(d) 0 2 3 5 4 1

(e) 0 1 3 5 6 2



2. Give pseudocode for an algorithm to check if a seqence S[0..(n-1)] for a graph G stored in an adjacency matrix A[0..(n-1)][0..(n-1)] gives a Hamilton cycle.

Assignment #5: Available from class web page. Due Friday Dec. 2 at the beginning of class.

Final exam tutorial:
Sunday Dec. 4 at 12:00pm

Introduction to NP-completeness.

The class of NP-complete problems is defined.

Satisfiability, the most famous NP-complete problem is introduced.

To prove new problems are undecidable, we use halting problem reductions.

To prove new problems are NP-complete, reductions are also used but the transformation must preserve polynomial running time complexity.

Table 1: Comparing polynomial and exponential time complexity.
Assume a problem of size one takes 0.000001 seconds (1 microsecond).

	Size *n*					
	10	**20**	**30**	**40**	**50**	**60**
n	0.00001 second	0.00002 second	0.00003 second	0.00004 second	0.00005 second	0.00006 second
n^2	0.0001 second	0.0004 second	0.0009 second	0.0016 second	0.0025 second	0.0036 second
n^3	0.001 second	0.008 second	0.027 second	0.064 second	0.125 second	0.216 second
n^5	0.1 second	3.2 second	24.3 second	1.7 minutes	5.2 minutes	13.2 minutes
2^n	0.001 second	1.0 second	17.9 minutes	12.7 days	35.7 years	366 centuries
3^n	0.059 second	58 minutes	6.5 years	3855 centuries	$2*10^8$ centuries	$1.3*10^{13}$ centuries

(from M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-completeness*, W. H. Freeman, New York, 1979.)

Table 2: Effect of improved technology on several polynomial and exponential time algorithms. The following table represents the size of the largest problem instance solvable in 1 hour.

Time Complexity function	With present computer	With computer 100 times faster	With computer 1000 times faster
n	N1	100 N1	1000 N1
n^2	N2	10 N2	31.6 N2
n^3	N3	4.46 N3	10 N3
n^5	N4	2.5 N4	3.98 N4
2^n	N5	N5+6.64	N5+9.97
3^n	N6	N6+4.19	N6+6.29

(from M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-completeness*, W. H. Freeman, New York, 1979.)

Class P

A decision problem (yes/no question) is *in the class P* if there is a polynomial time algorithm for solving it.

Polynomial time: $O(n^c)$ for some constant c.

If a problem is solvable in polynomial time for

- some sensible encoding of the input
- some reasonable machine (TM/RAM/PC)

it can be solved in polynomial time for all other sensible encodings/reasonable machines.

A program that runs on a Random Access Machine (such as our modern day computers) in time T(n) can be simulated on a single tape Turing machine in time $O(T^3(n))$.

This distinction is made between polynomial time and anything else (sometimes referred to as exponential time) because of the blowup in computation times which appear for exponential algorithms.

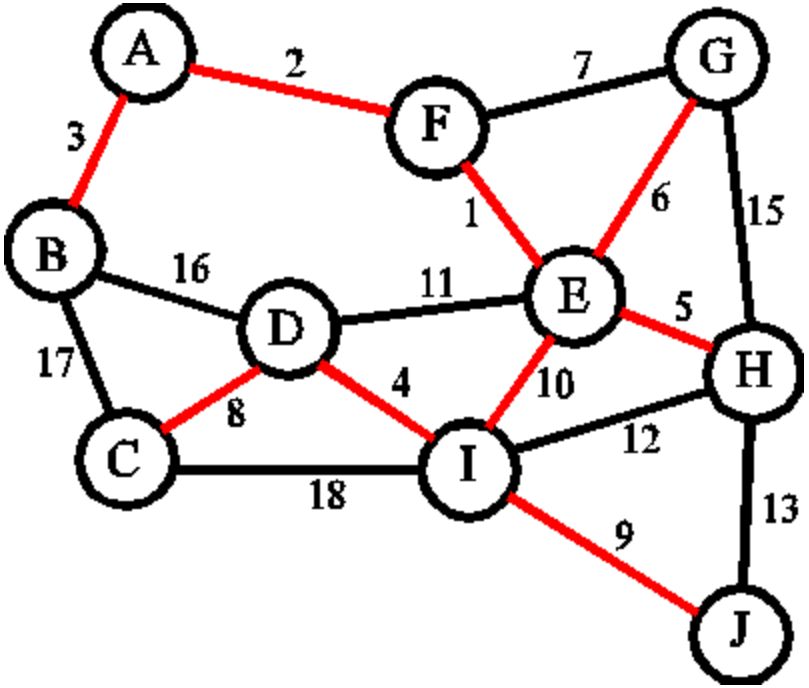
Example problem which is in P:

Minimum Weight Spanning Tree (CSC 225).

Input: Graph G, integer k.

Question: Does G have a spanning tree of weight at most *k*?

If you are provided with a tree with weight at most k as part of the solution, the answer can be verified in $O(n^2)$ time.



Class NP

A decision problem (yes/no question) is in the *class NP* if it has a nondeterministic polynomial time algorithm. Informally, such an algorithm:

1.Guesses a solution (nondeterministically).

2. Checks deterministically in polynomial time that the answer is correct.

Or equivalently, when the answer is "yes", there is a certificate (a solution meeting the criteria) that can be verified in polynomial time (deterministically).

Hamilton Cycle is in NP: Input graph G.

Does G have a Ham. cycle?

Certificate:

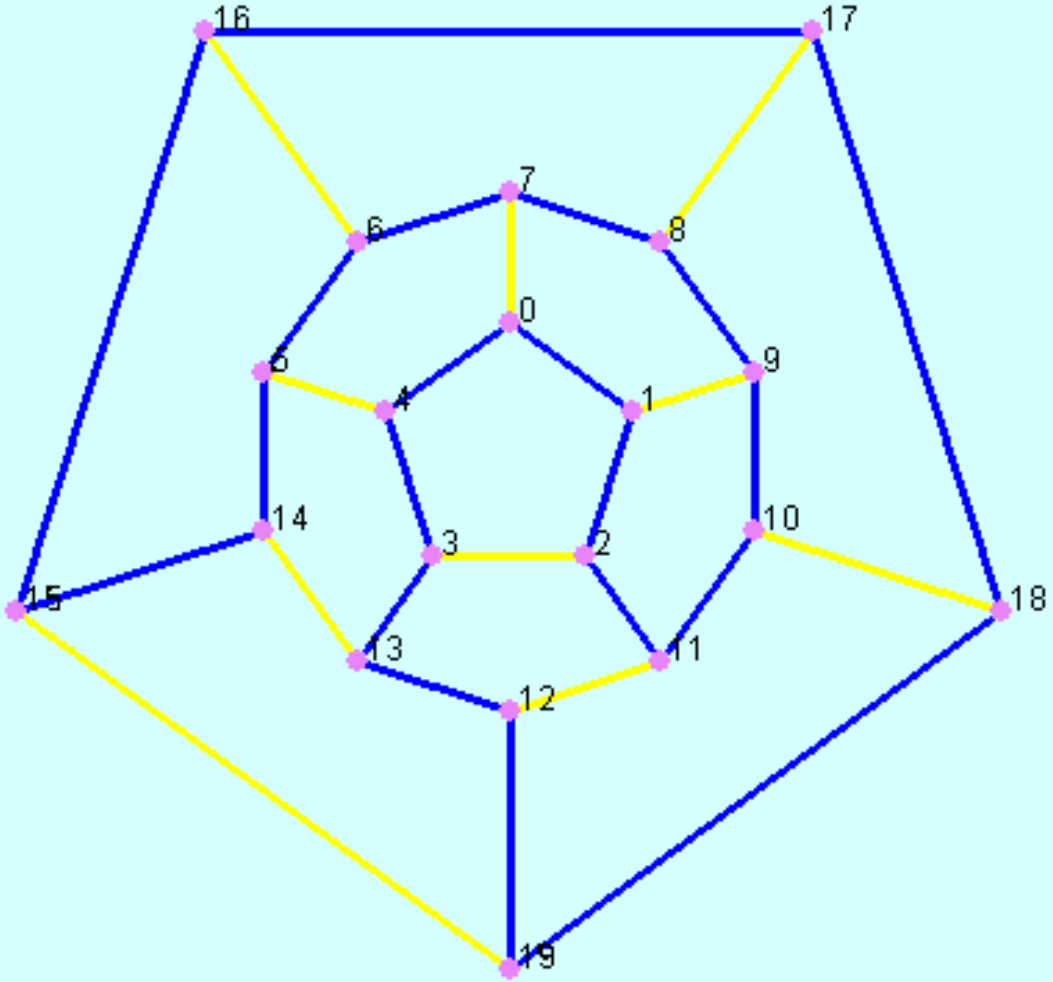
0, 1, 2, 11, 10,

9, 8, 7, 6, 5,

14, 15, 16, 17,

18, 19, 12, 13,

3, 4

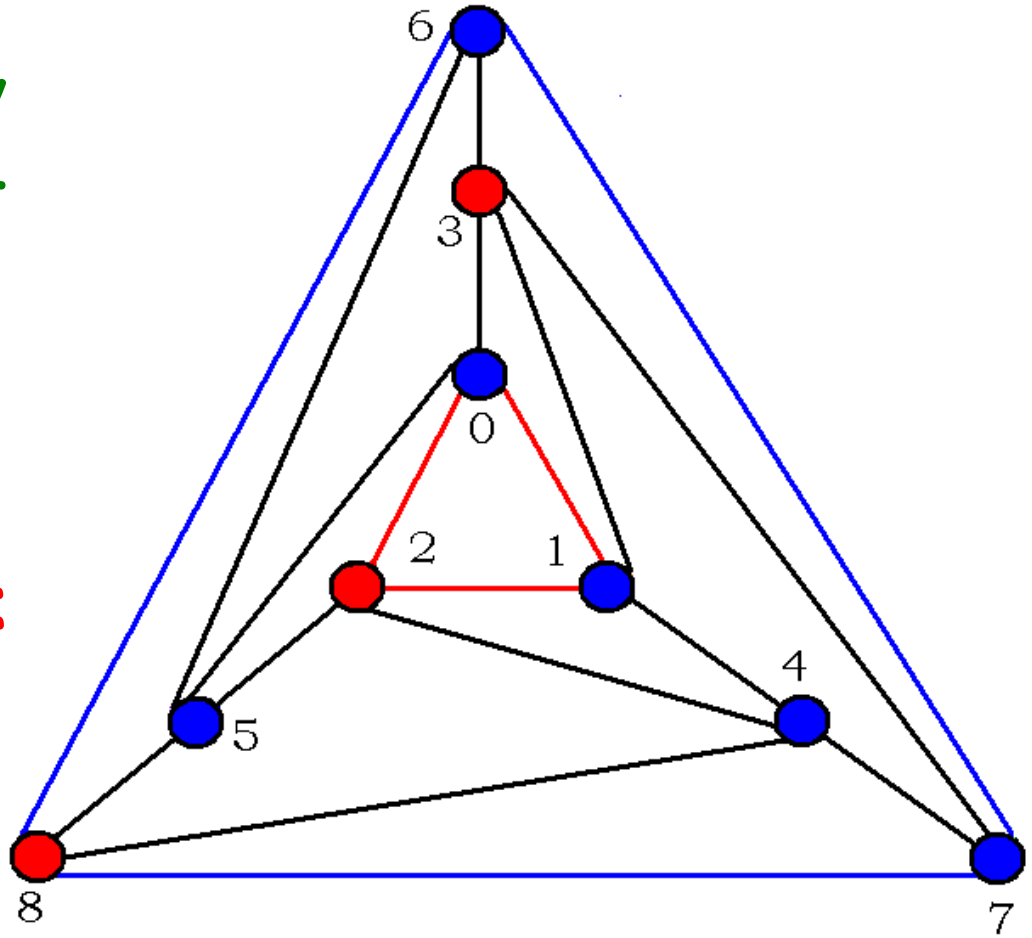


Independent Set is in NP: Given a graph G and integer k, does G have an independent set of order k?

Vertices u and v are independent if edge (u, v) is not in G.

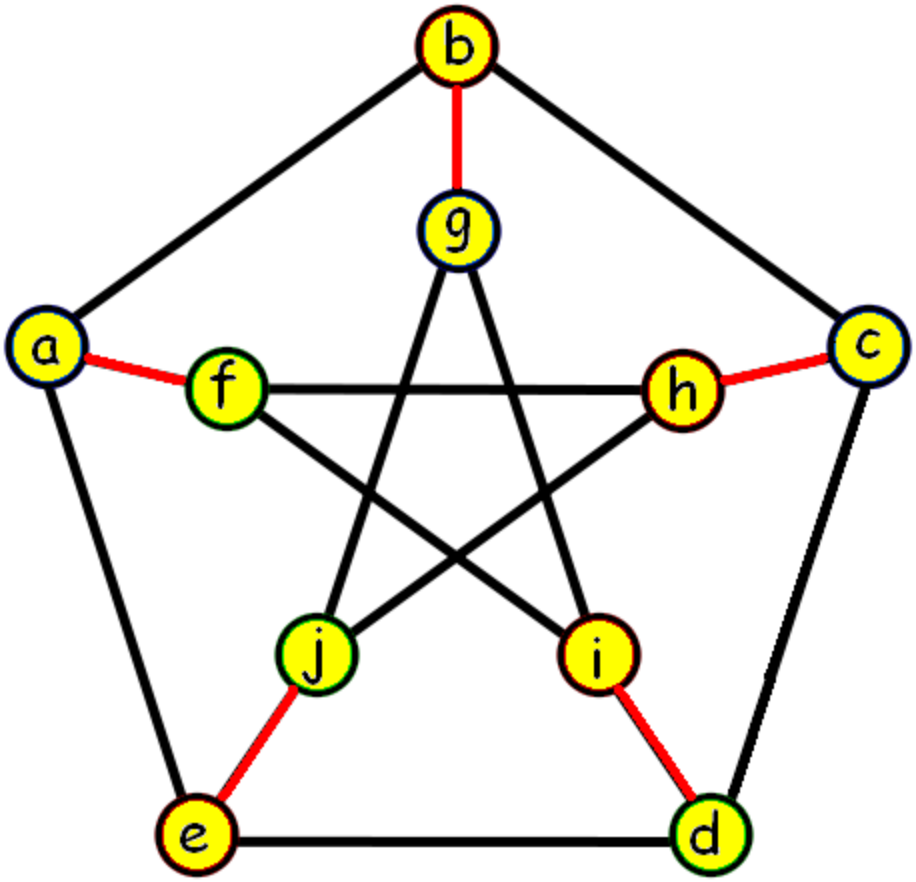
Certificate, k=3:

2, 3, 8



Matching is in NP: Given a graph G and integer k, does G have a k-edge matching?

Matching:
disjoint
edges.



Certificate, k=5:

(a,f) (b,g) (c,h)

(d,i)(e,j)

Does P= NP? the Clay Mathematics Institute has offered a $1 million US prize for the first correct proof.

Some problems in NP not known to be in P:

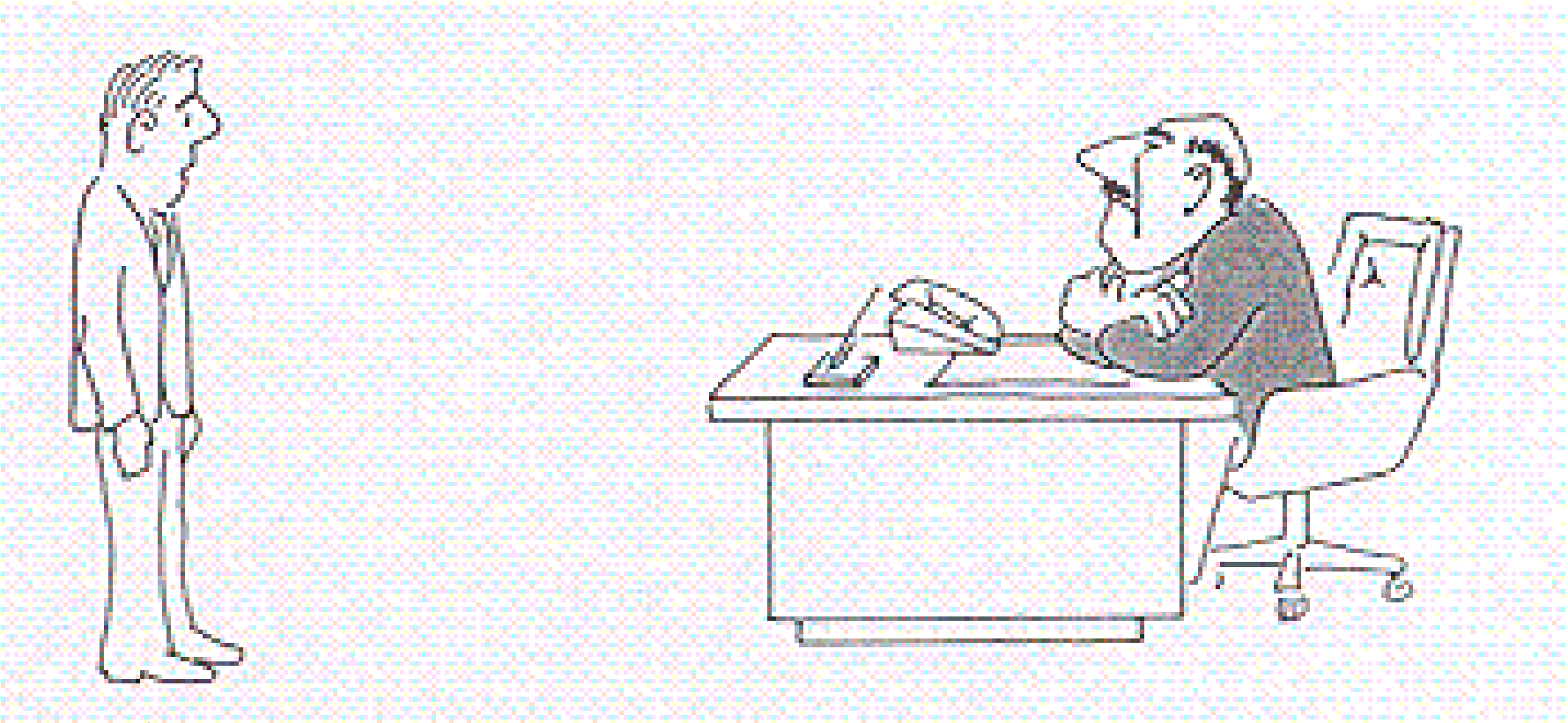
Hamilton Path/Cycle

Independent Set

Satisfiability

Note: Matching is in P. Learn more in a graph algorithms class.

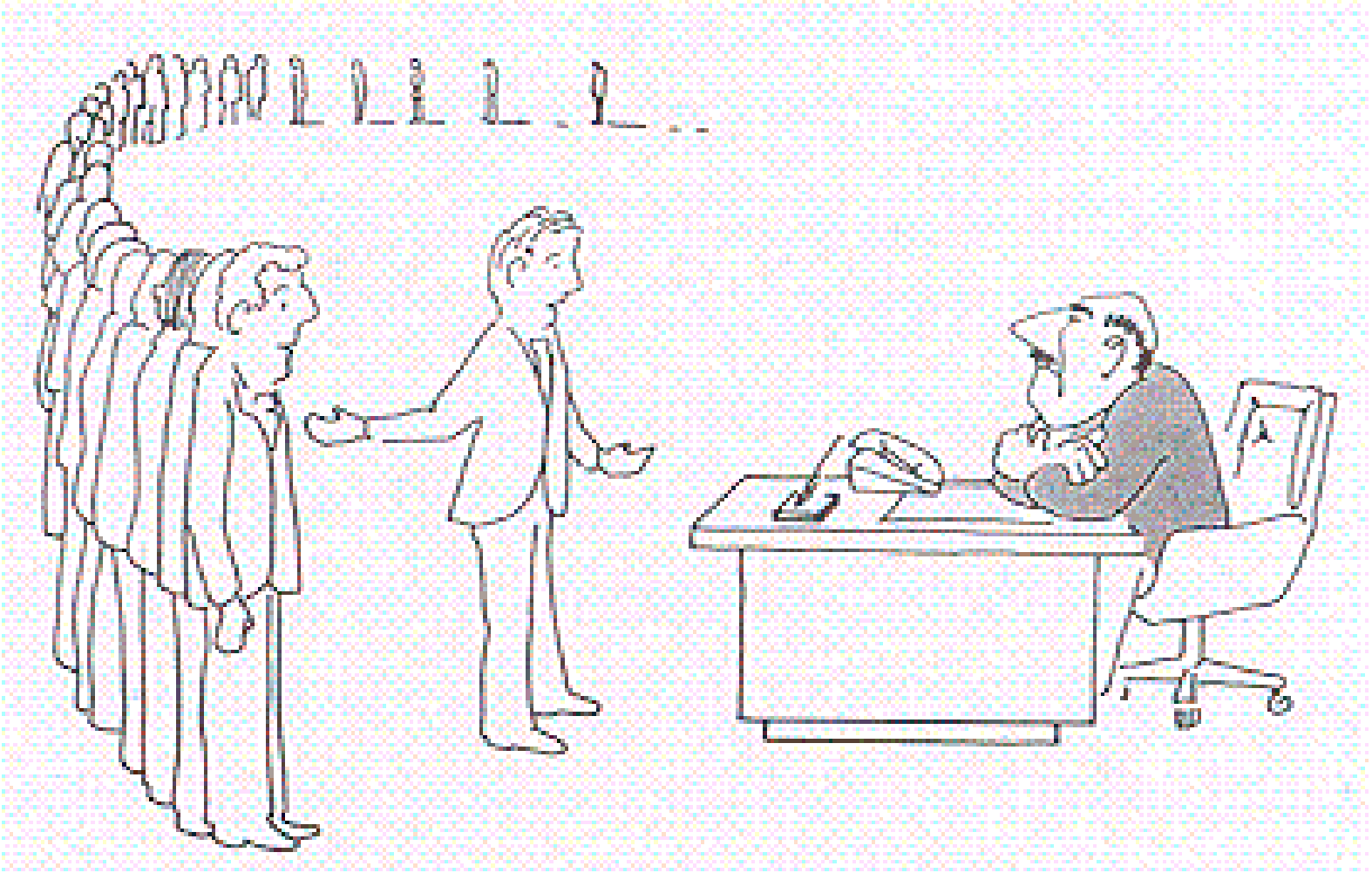
NP-completeness



I can't find an efficient algorithm,

I guess I'm just too dumb.

I can't find an efficient algorithm, because no such algorithm is possible.



I can't find an efficient algorithm, but neither can all these famous people.

NP-complete Problems

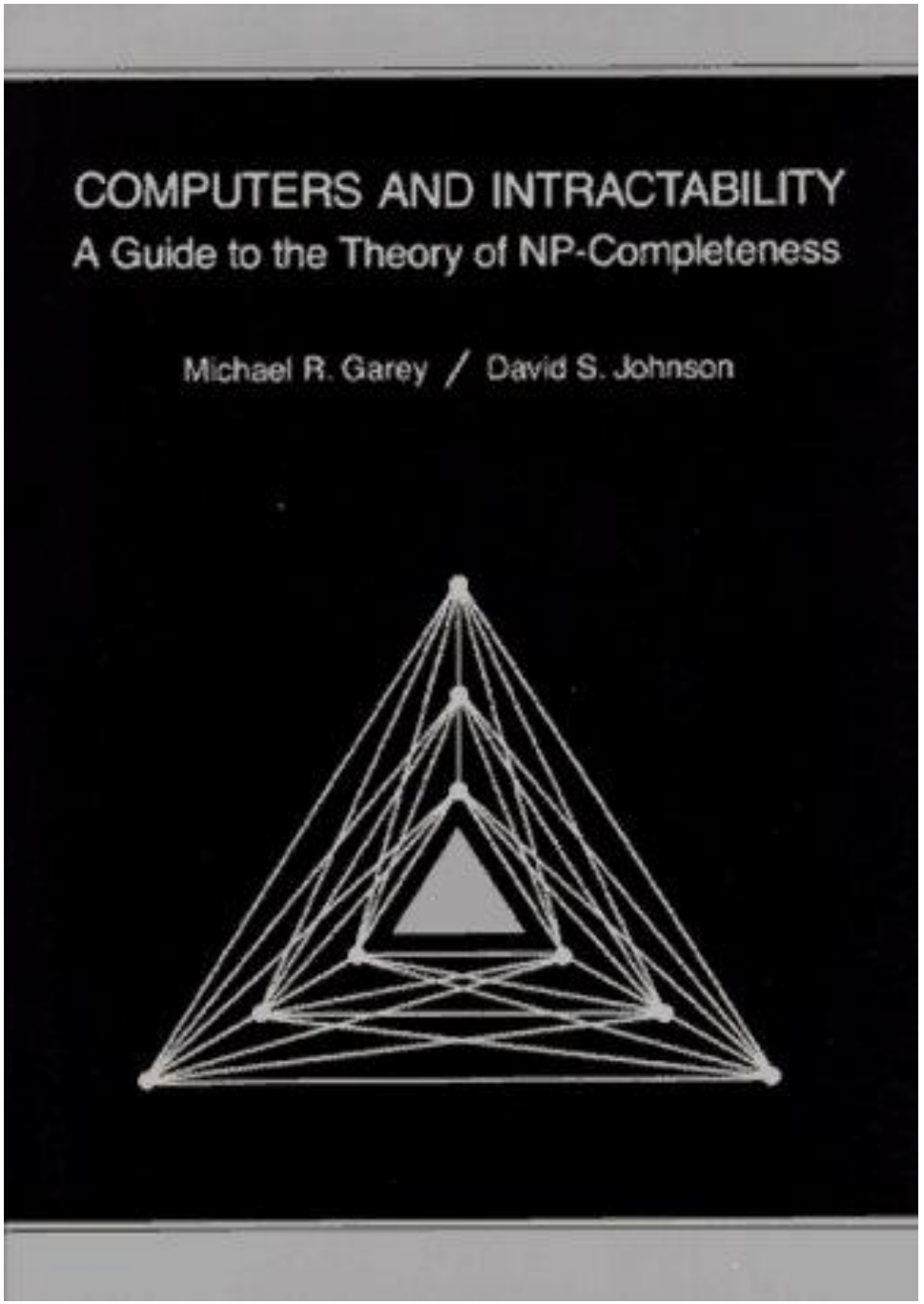
The class of problems in NP which are the "hardest" are called the *NP-complete* problems.

A problem Q in NP is NP-complete if the existence of a polynomial time algorithm for Q implies the existence of a polynomial time algorithm for all problems in NP.

Steve Cook in 1971 proved that SAT is NP-complete. Proof: will be given in our last class. Other problems: use reductions.

Bible for NP-completeness:

M. R. Garey and D. S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completness*, W. H. Freeman, 1st ed. (1979).



A problem Q in NP is NP-complete if the existence of a polynomial time algorithm for Q implies the existence of a polynomial time algorithm for all problems in NP.

SAT (Satisfiability)

Variables: $u_1, u_2, u_3, \ldots u_k$.

A **literal** is a variable u_i or the negation of a variable $\neg u_i$.

If u is set to *true* then ¬ u is *false* and if u is set to *false* then ¬ u is *true*.

A **clause** is a set of literals. A clause is *true* if at least one of the literals in the clause is *true*.

The input to SAT is a collection of clauses.

This SAT problem has solution
u_1=T, u_2=F, u_3= T, u_4=F

(u_1 OR u_2 OR u_4) AND (¬ u_2 OR u_4) AND
(¬ u_1 OR u_3) AND (¬ u_4 OR ¬ u_1)

Does this SAT problem have a solution?

(u_1 OR u_2) AND (¬ u_2 OR u_3) AND

(¬ u_3 OR ¬ u_1) AND (¬ u_2 OR ¬ u_3) AND

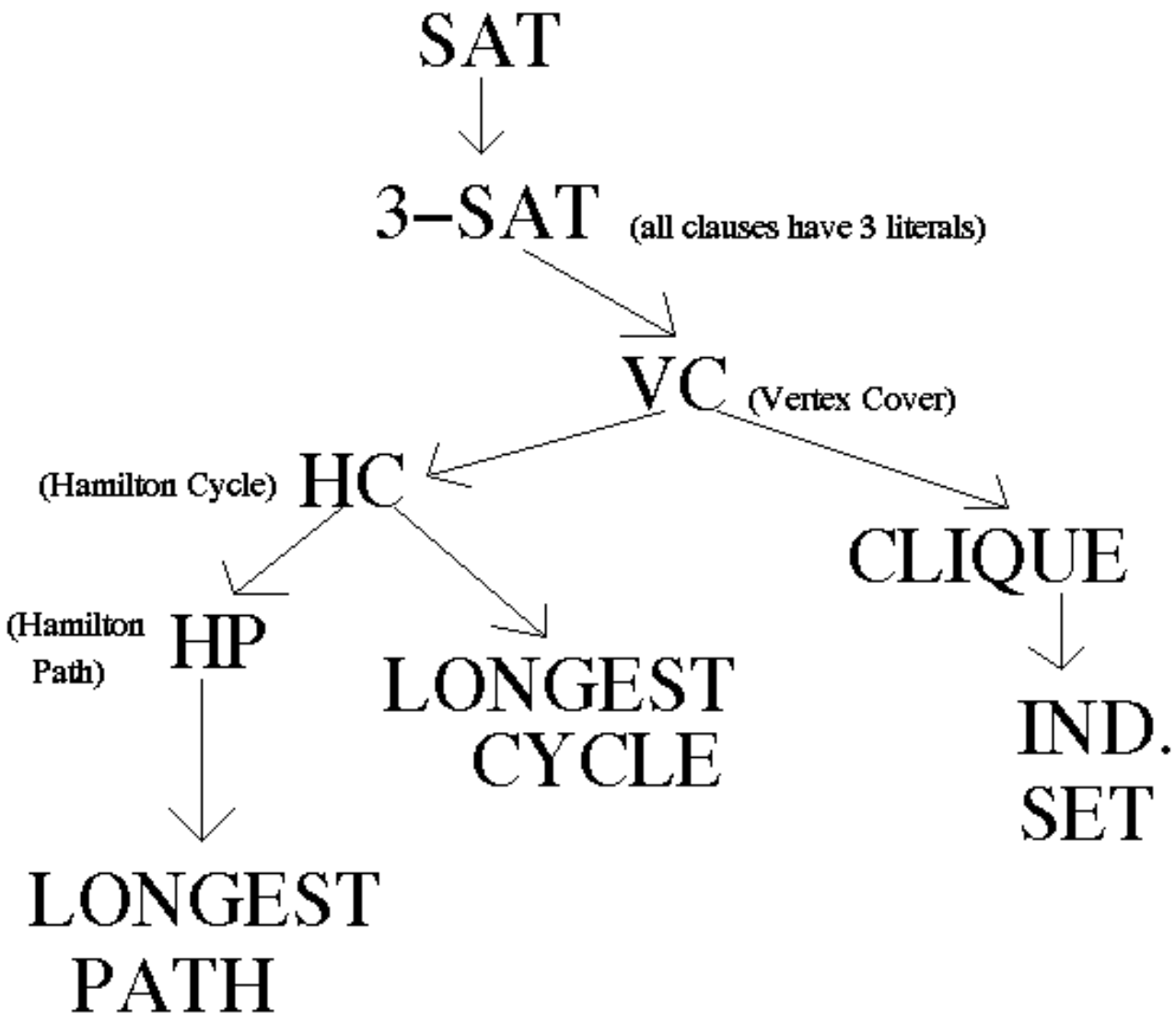
(u_3 OR ¬ u_1)

SAT (Satisfiability)

The output is the answer to: Is there an assignment of *true/false* to the variables so that every clause is satisfied (**satisfied** means the clause is *true*)?

If the answer is yes, such an assignment of the variables is called a **truth assignment**.

SAT is in NP: Certificate is true/false value for each variable in satisfying assignment.



History of NP−completeness Reductions