The n-Queen graph: Vertices- squares on an n by n chessboard. Two vertices are adjacent if a queen could move from one square to another. That is, if the squares are on the same row, column, diagonal or back diagonal. Red vertices: dominating set vertices.

- 1. How many automorphisms of the graph map each dominating set to itself?
- 2. How many different dominating sets does each one correspond to?









Annoucements

The slides I originally posted had some errors in the pictures that have been corrected.

- Some graph classes that may have polynomial time algorithms for hard problems:
- planar, perfect, k-trees, chordal, permutation, interval, circular arc, intersection, bounded tree width, bounded facewidth, claw-free, fullerenes, benzenoids,

The "on-time" deadline for the literature review has been extended until 11:55pm on Tuesday Oct. 28. The late submission deadline of Fri. Oct. 31 by 11:55pm is not changed. Upload submissions to connex.

The queen's graph is not a regular graph.







Red cell dominates 25 vertices. Red cell dominates 23 vertices. Red cell dominates 19 vertices.

Number of cells each cell dominates:



Our naïve approach suggests that two cells can dominate 25 + 25= 50 cells.

But if we sort these numbers in reverse order: 25, 23, 23, ...

We see that 2 cells could dominate at most 25 + 23 = 48 vertices/cells.

Choose center cell to be in dominating set:

19	19	19	19	19	19	19
19	21	21	21	21	21	19
19	21	23	23	23	21	19
19	21	23	25	23	21	19
19	21	23	23	23	21	19
19	21	21	21	21	21	19
19	19	19	19	19	19	19

8	11	9	8	9	11	8
11	10	11	8	11	10	11
9	11	10	12	10	11	9
8	8	12		12	8	8
9	11	10	12	10	11	9
11	10	11	8	11	10	11
8	11	9	8	9	11	8

Yellow squares are dominated.

The number of undominated cells each cell would dominate decreases dramatically. New sorted order: 12, 12, 12, 12, 10, 10 ... 24 cells are not dominated: need at least 2 more $(12 + 12 \ge 24)$. Greedy approach: choose a cell dominating a maximum number of undominated vertices:



New sorted order: 8, 7, 7, 6, 6, ... 12 cells are not dominated: need at least 2 more $(8+7 \ge 12)$. If we backtrack and color center cell blue:

19	19	19	19	19	19	19
19	21	21	21	21	21	19
19	21	23	23	23	21	19
19	21	23	25	23	21	19
19	21	23	23	23	21	19
19	21	21	21	21	21	19
19	19	19	19	19	19	19

New sorted order: 23, 23, 23, ... 49 cells are not dominated: need at least 3 more $(23 + 23 + 23 \ge 49)$. Our simple formula for an upper bound: $n_extra \ge num_not_dominated / (\Delta + 1)$ is very easy to compute: O(1) time.

It takes more time to maintain a better bound. One tactic: maintain for k= 0, 1, 2, ..., n the number of white vertices dominating k undominated vertices (num[0...n]).

x= maximum number of undominated vertices some white vertex dominates.

Go from x downwards adding on for one vertex at a time the number it might newly dominate until the total is at least the number of undominated vertices.

```
n_extra=0; sum=0;
for (k = x; k \ge 1; k - -)
   for (j=0; j < num[k]; j++)</pre>
       n_extra++;
      sum+= k:
       if (sum > num_not_dominated)
             goto check_bound;
// white vertices can't dominate all the rest.
return(0);
```

check_bound:;
if (size + n_extra >= min_size) return(0);