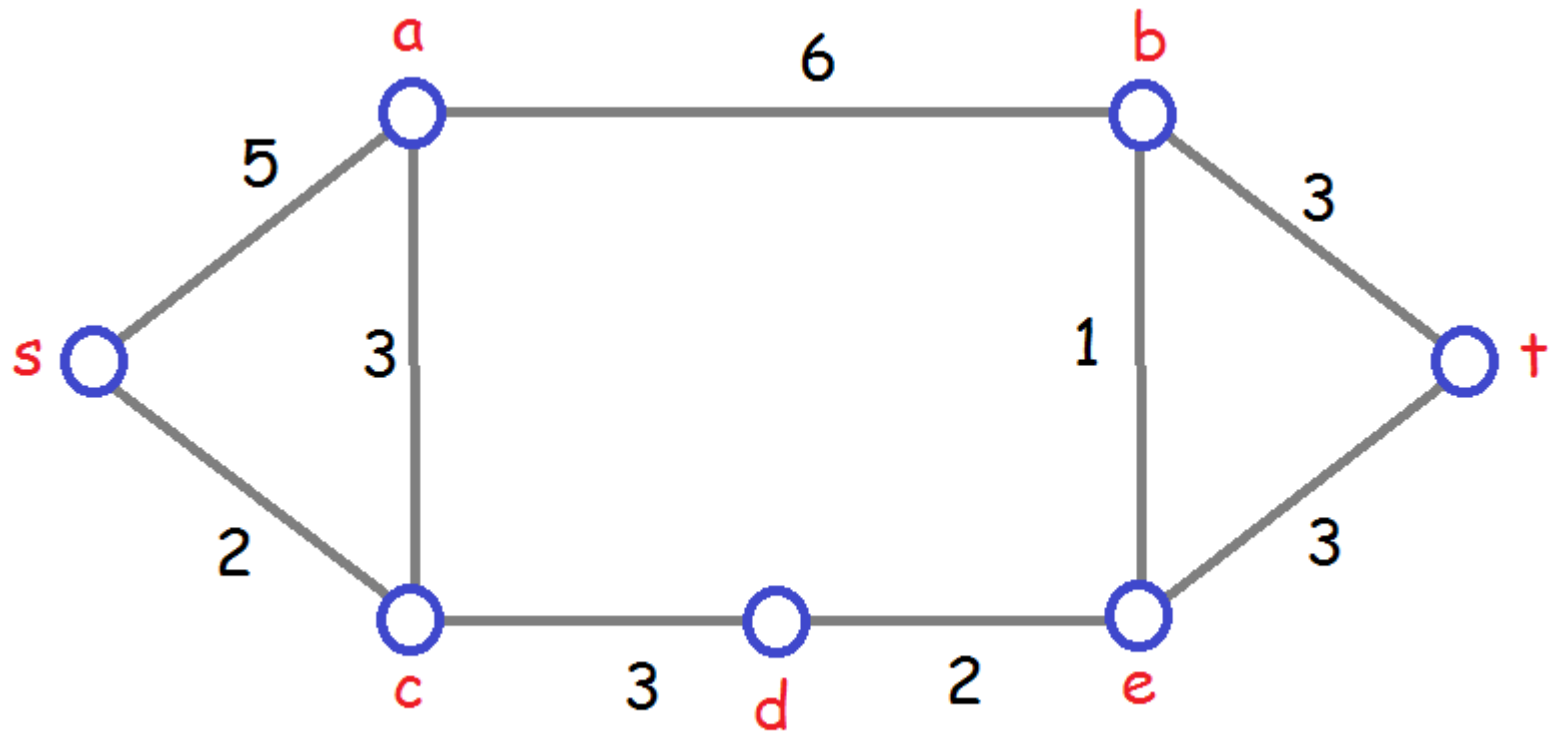


Use the Edmonds-Karp algorithm to find a maximum  $s,t$ -flow then indicate the resulting minimum cut.

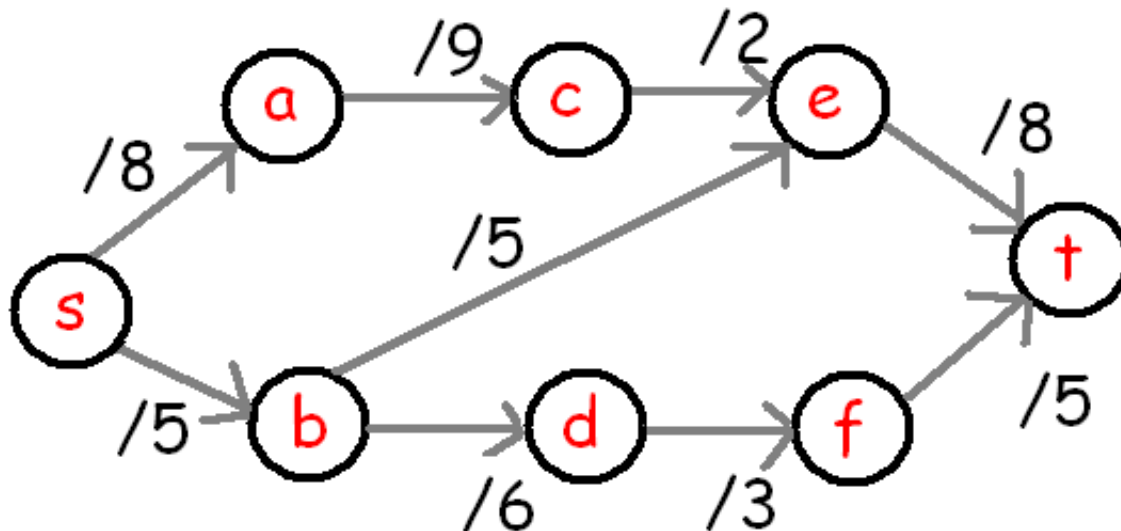


# Some applications of the minimum-cut maximum-flow algorithm:

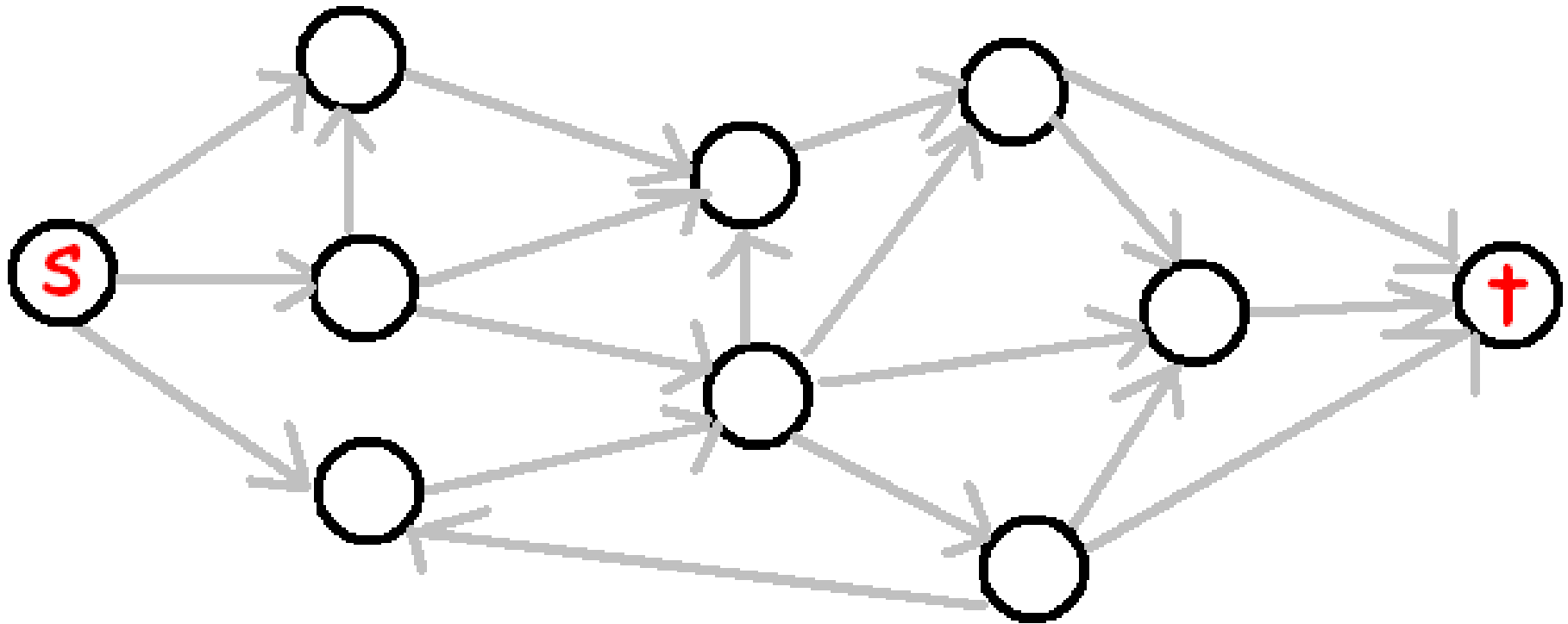
Vertex connectivity

Maximum matching in bipartite graphs

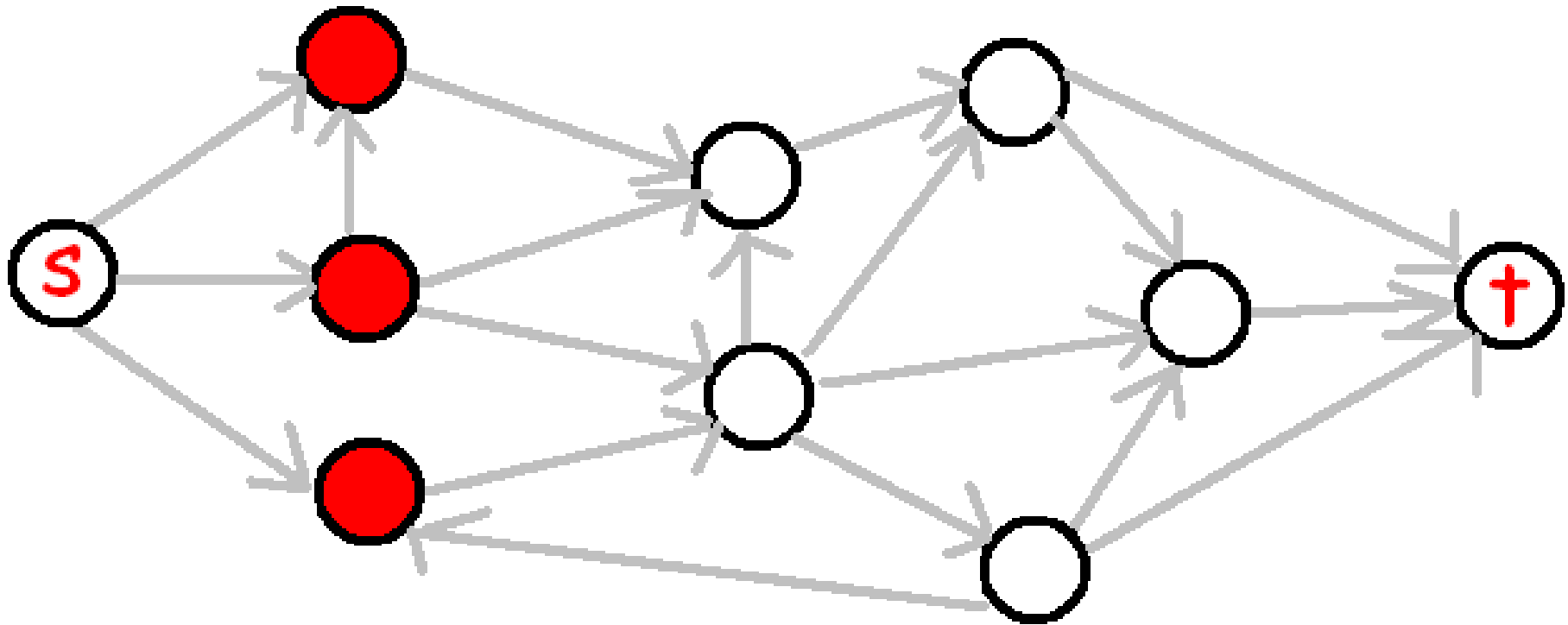
Minimum cut between each pair of nodes in an undirected graph



An **s,t-cut**: subset  $S$  of the vertices not including  $s$  or  $t$  so that  $G-S$  has no directed paths from  $s$  to  $t$ .

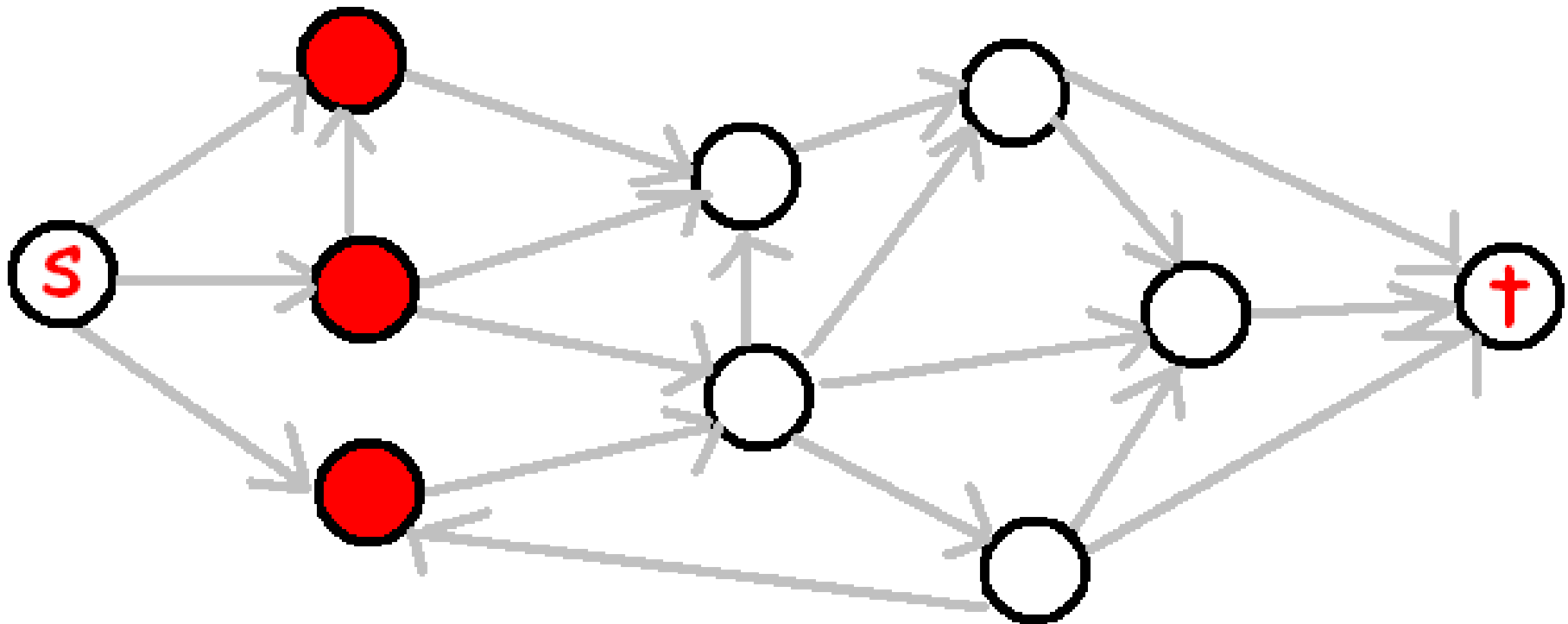


An **s,t-cut**: subset  $S$  of the vertices not including  $s$  or  $t$  so that  $G-S$  has no directed paths from  $s$  to  $t$ .



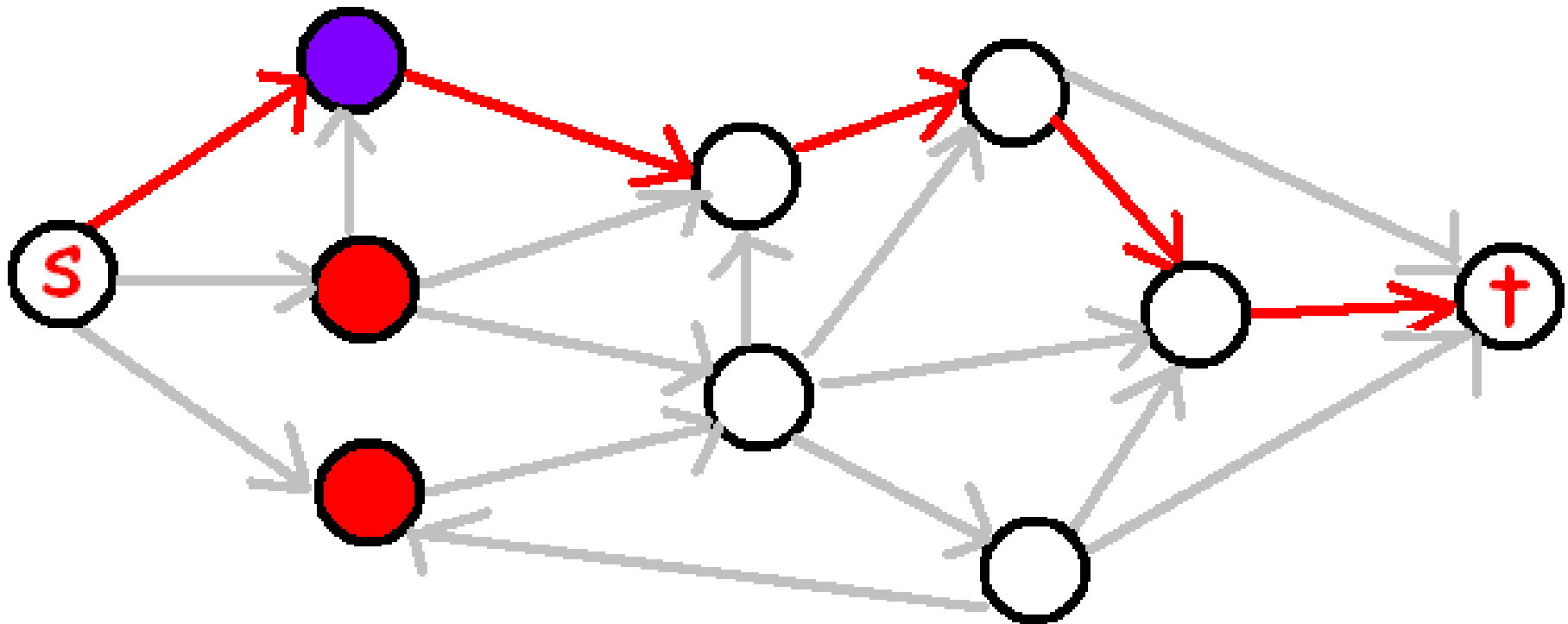
An **s,t-cut**: subset  $S$  of the vertices not including  $s$  or  $t$  so that  $G-S$  has no directed paths from  $s$  to  $t$ .

A minimal vertex cut:



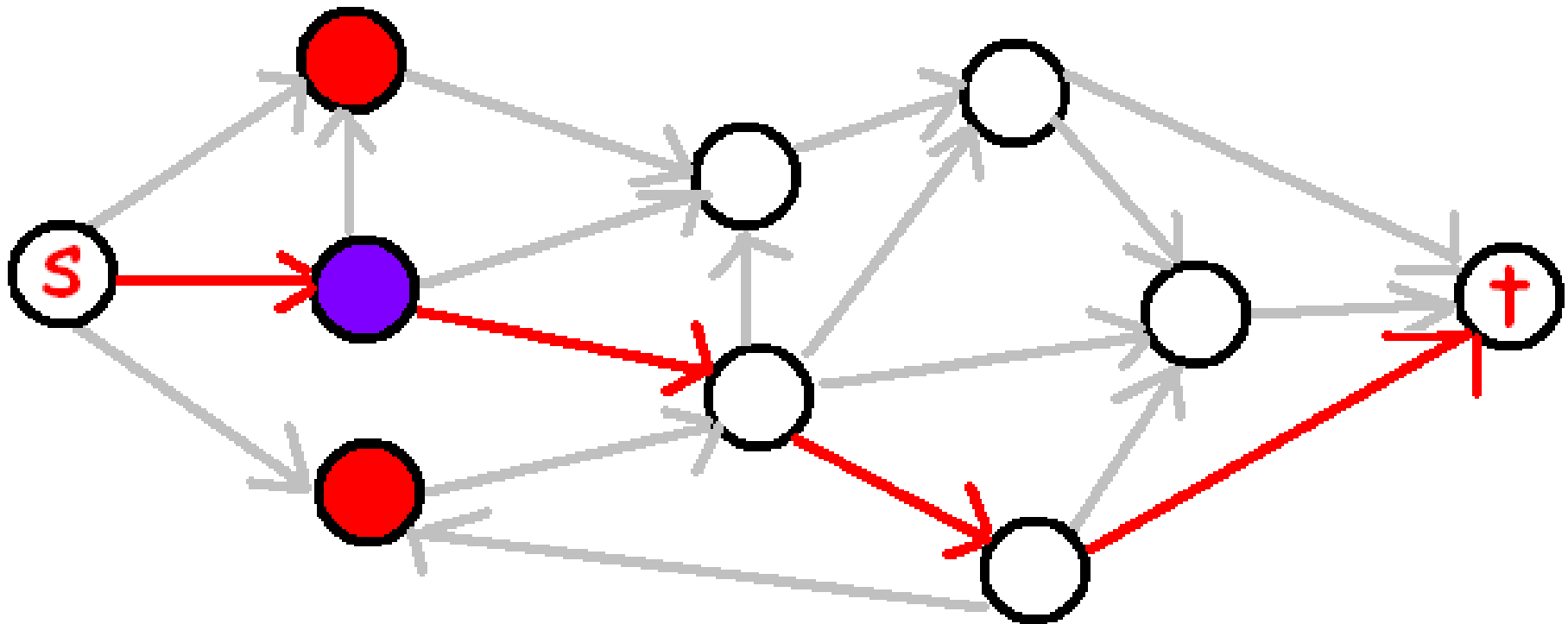
An **s,t-cut**: subset  $S$  of the vertices not including  $s$  or  $t$  so that  $G-S$  has no directed paths from  $s$  to  $t$ .

A minimal vertex cut:



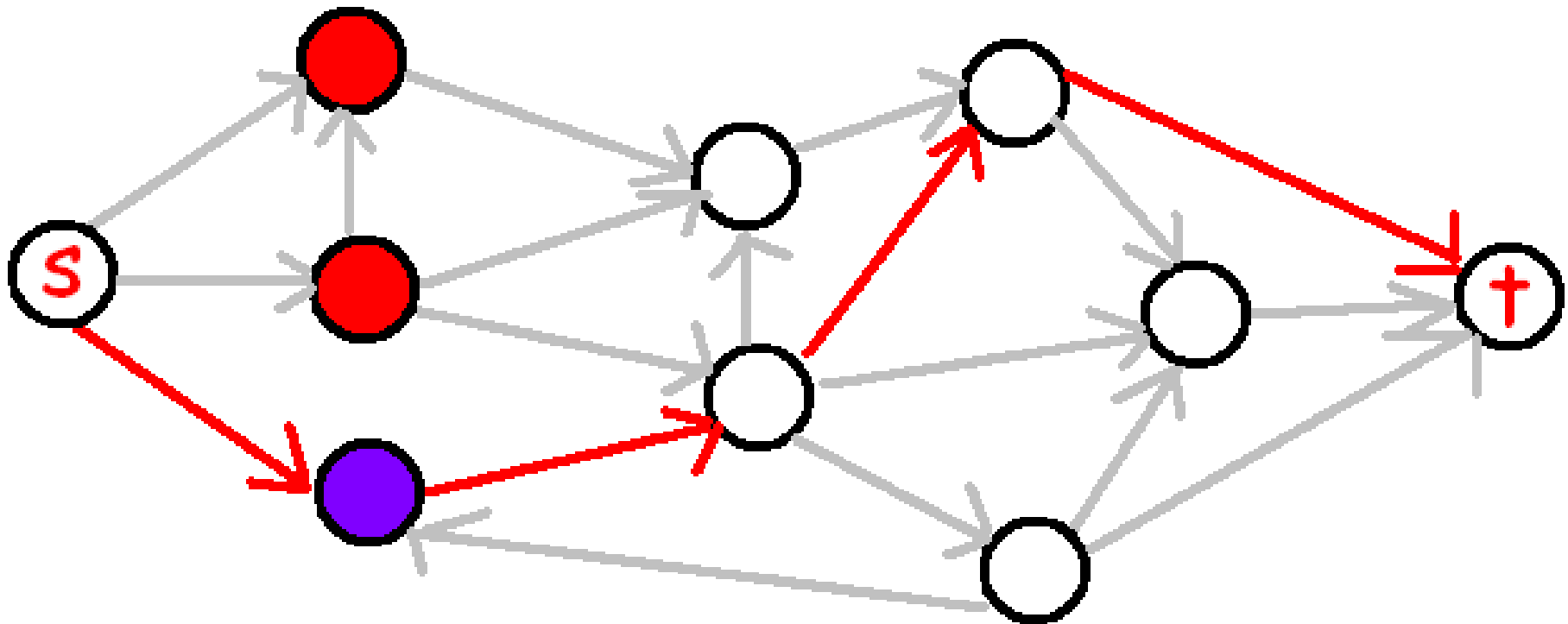
An **s,t-cut**: subset  $S$  of the vertices not including  $s$  or  $t$  so that  $G-S$  has no directed paths from  $s$  to  $t$ .

A minimal vertex cut:



An **s,t-cut**: subset  $S$  of the vertices not including  $s$  or  $t$  so that  $G-S$  has no directed paths from  $s$  to  $t$ .

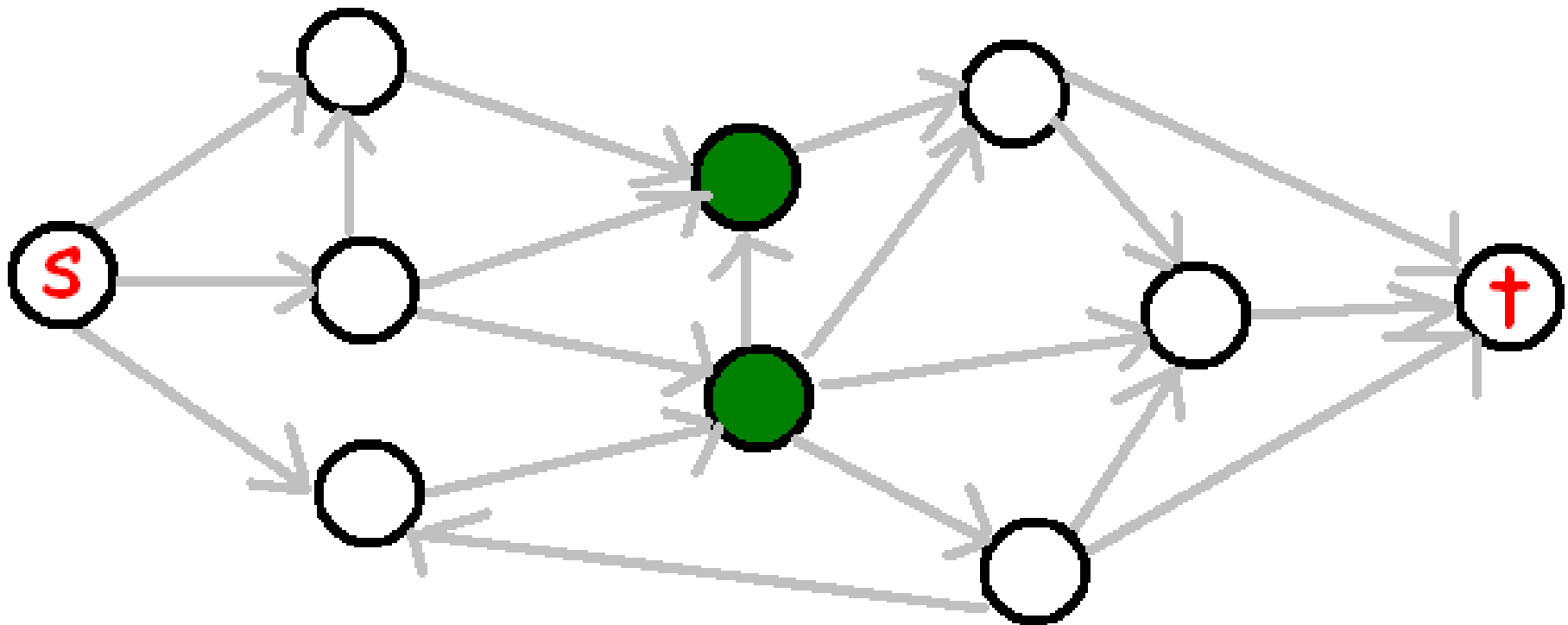
A minimal vertex cut:





An **s,t-cut**: subset  $S$  of the vertices not including  $s$  or  $t$  so that  $G-S$  has no directed paths from  $s$  to  $t$ .

A **MINIMUM** vertex cut:



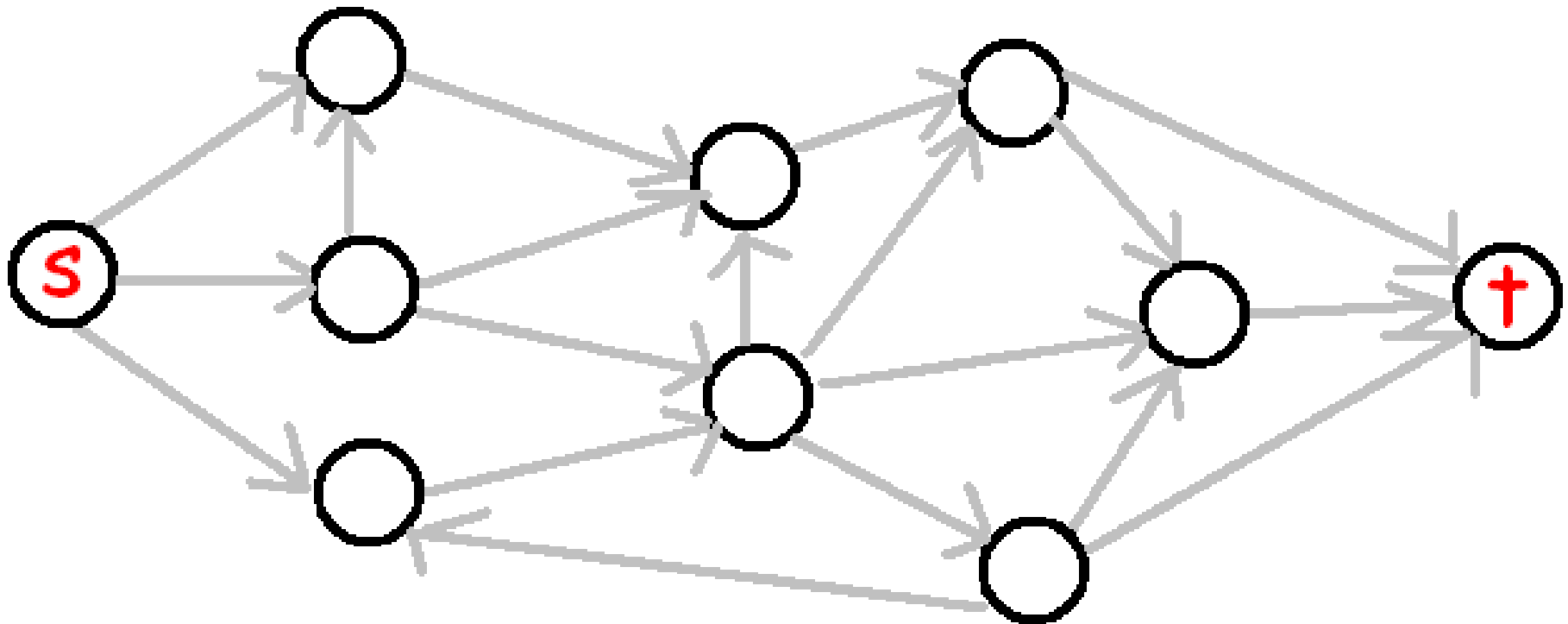
Given a directed graph  $G$ , source  $s$ , and sink  $t$ , find a minimum vertex cut between  $s$  and  $t$ .

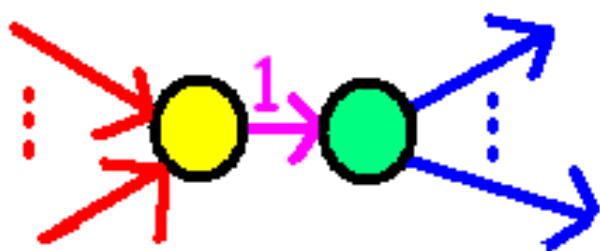
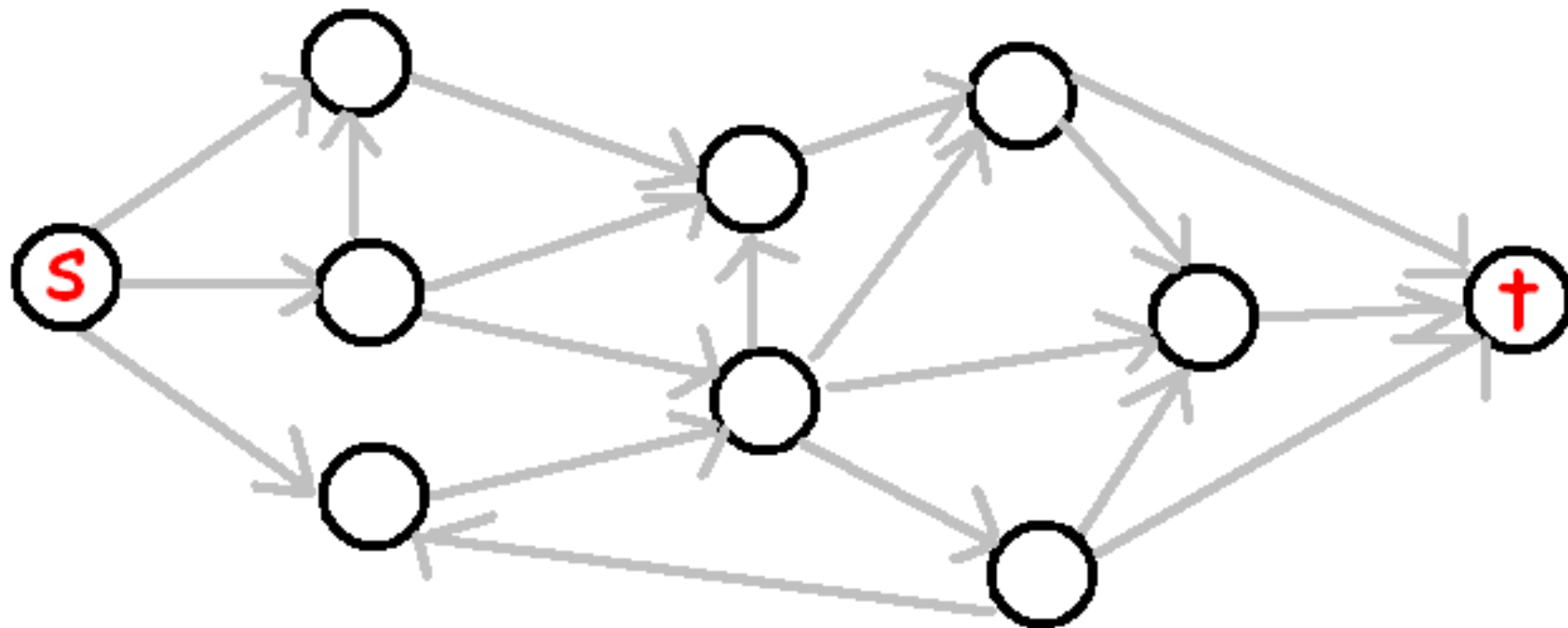
Undirected graph:



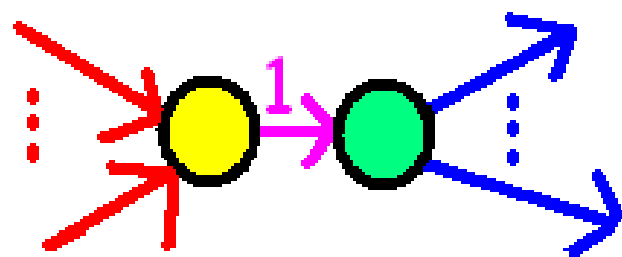
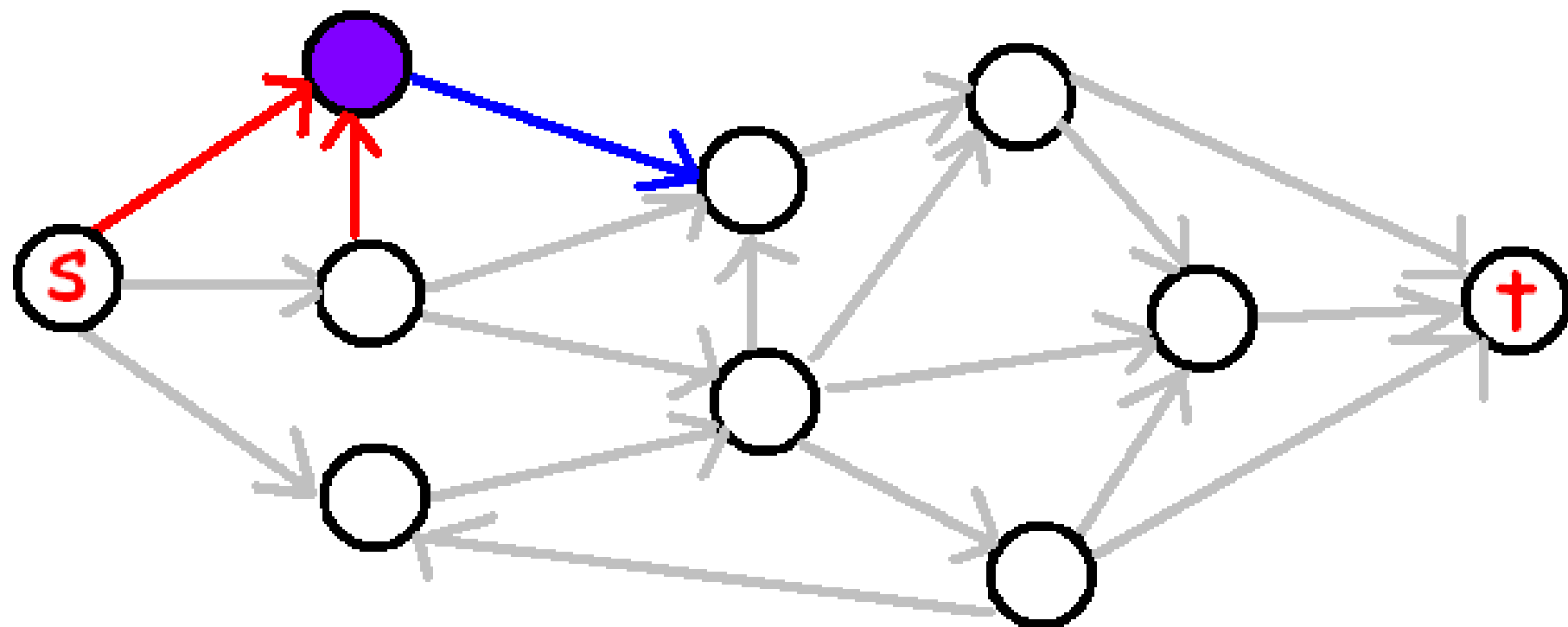
**Vertex-connectivity** = Minimum over all pairs  $s$  and  $t$  of the  $s, t$ -vertex connectivity in the corresponding directed graph.

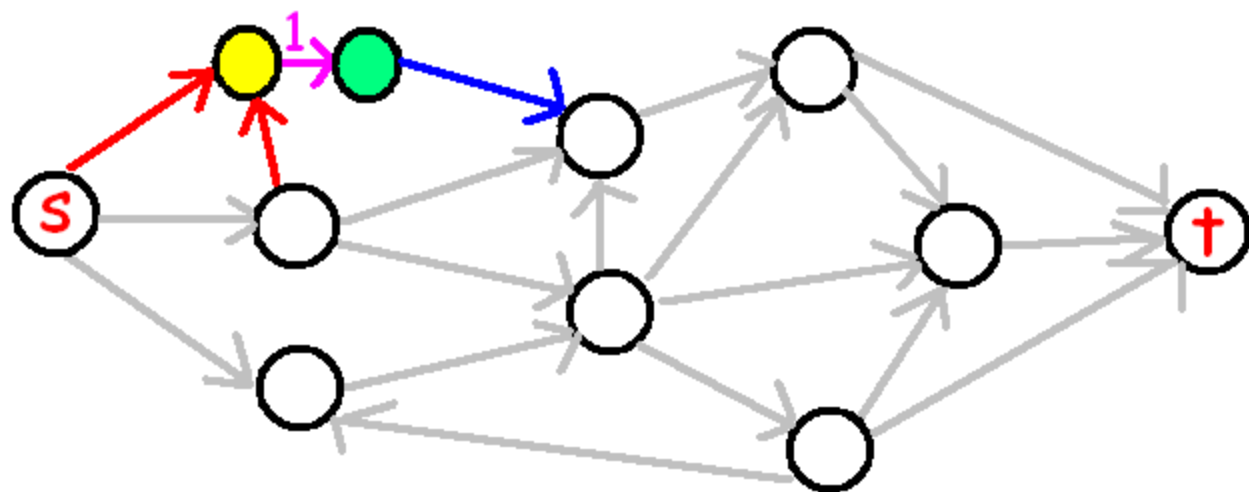
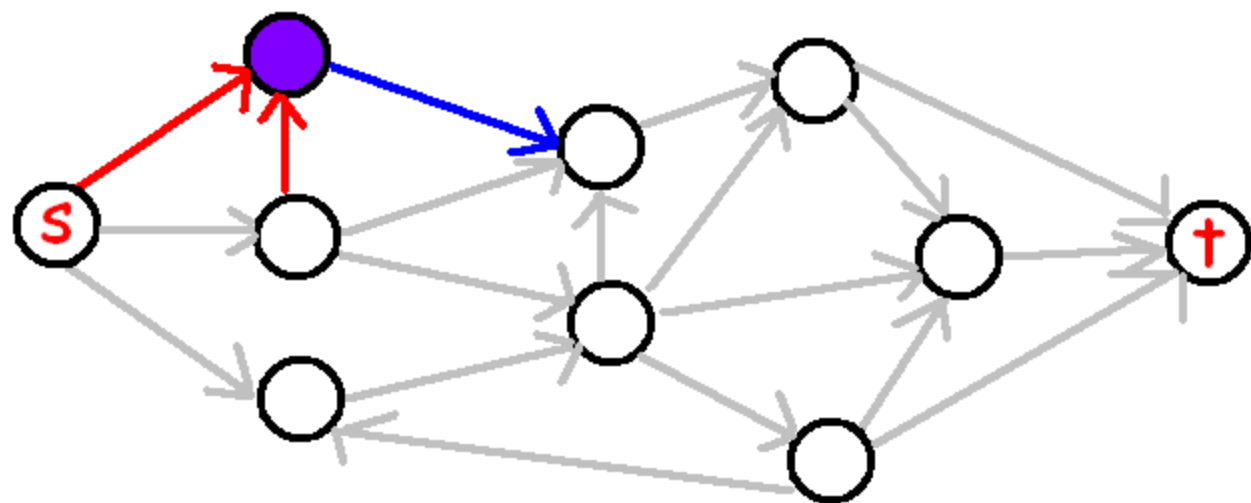
We can use min-cut max-flow to find a minimum vertex cut by first changing the network.

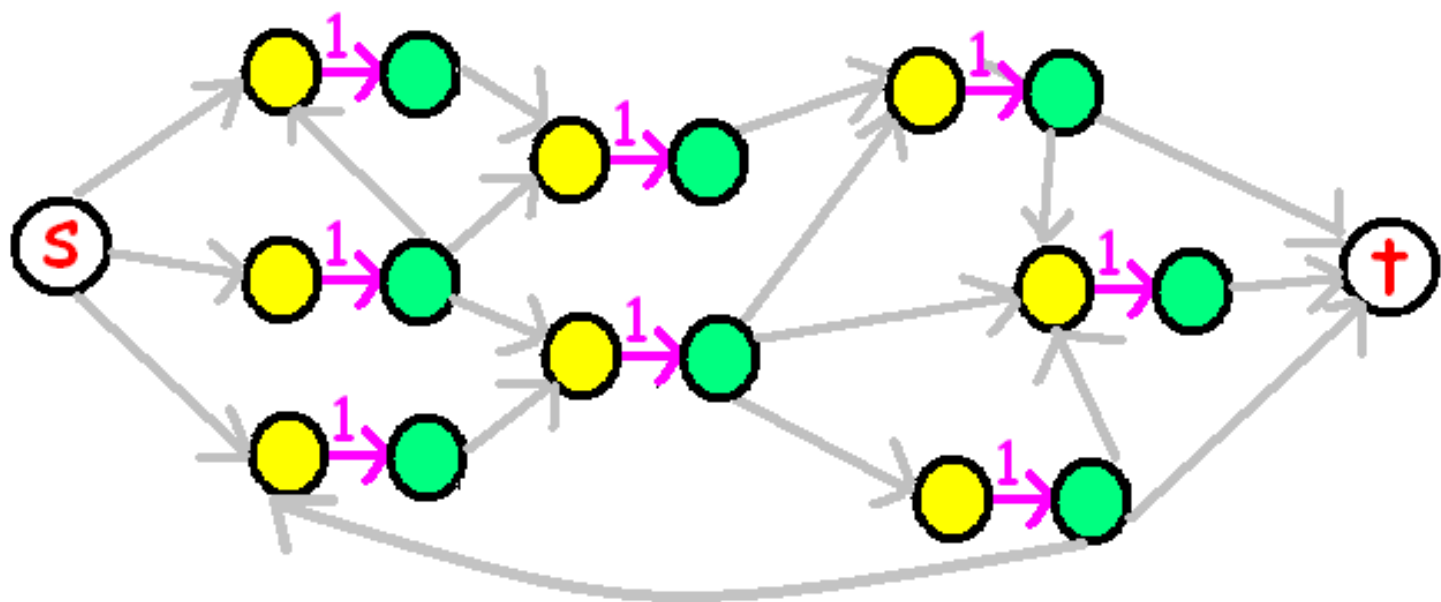
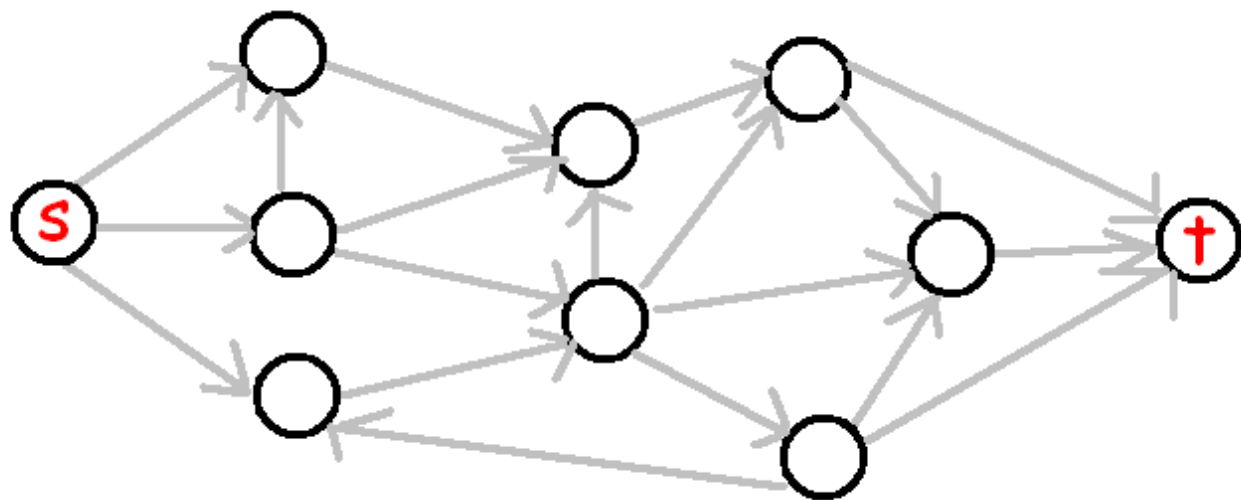


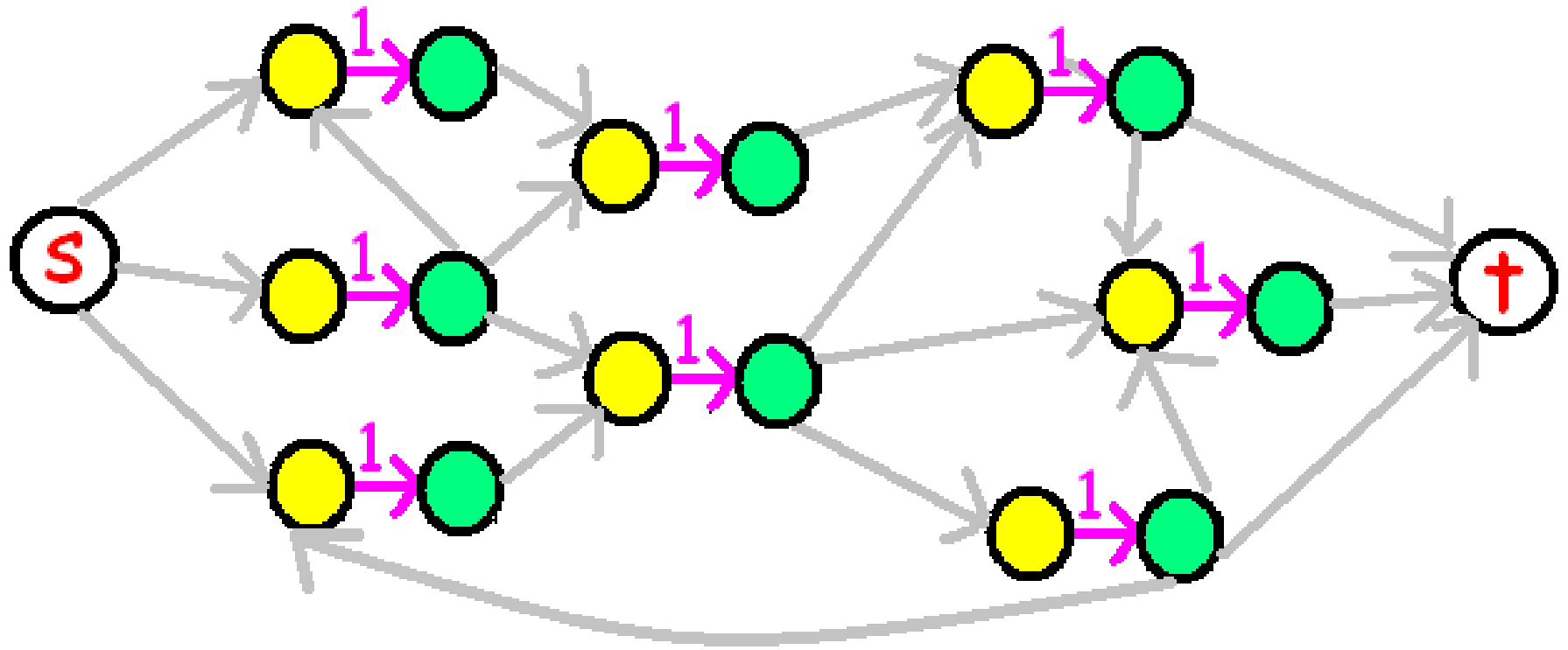


Gadget to  
replace  
vertices (but  
not s or t)







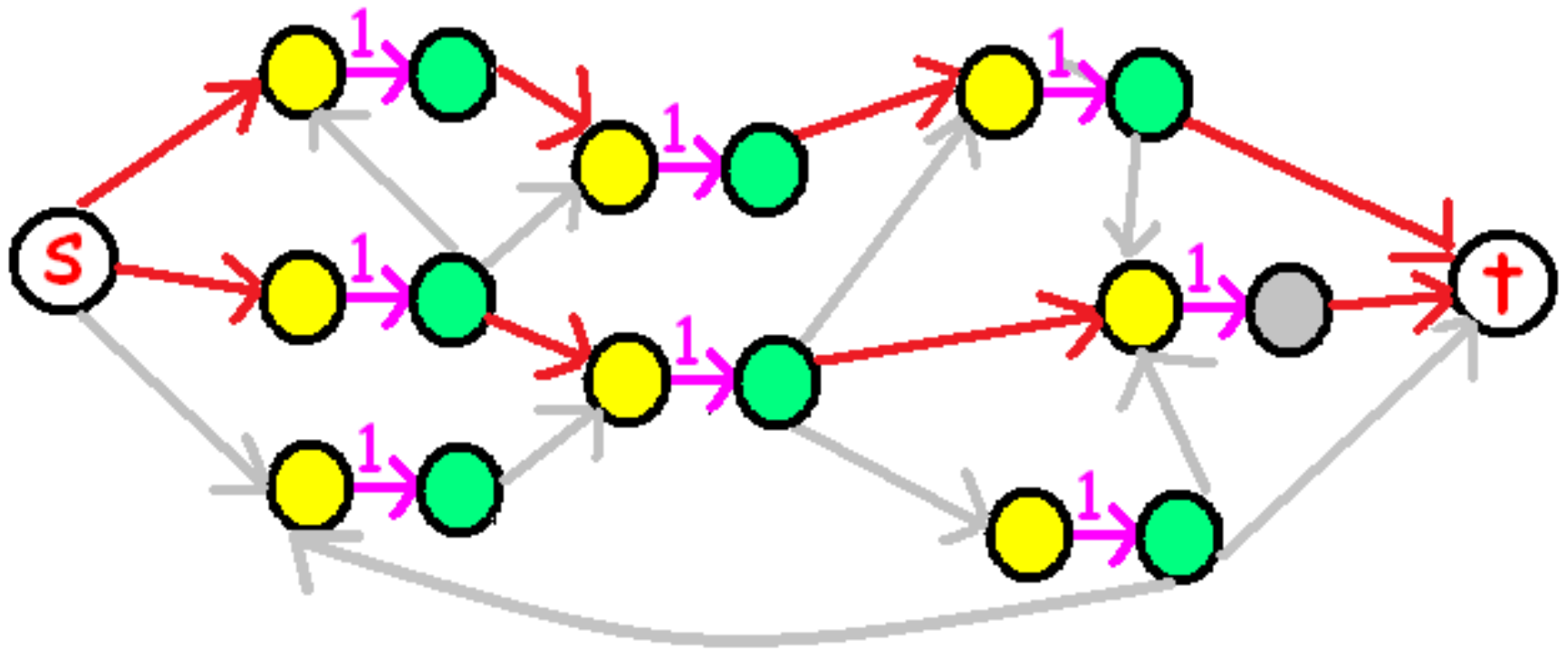


Directed Graph for maximum flow.

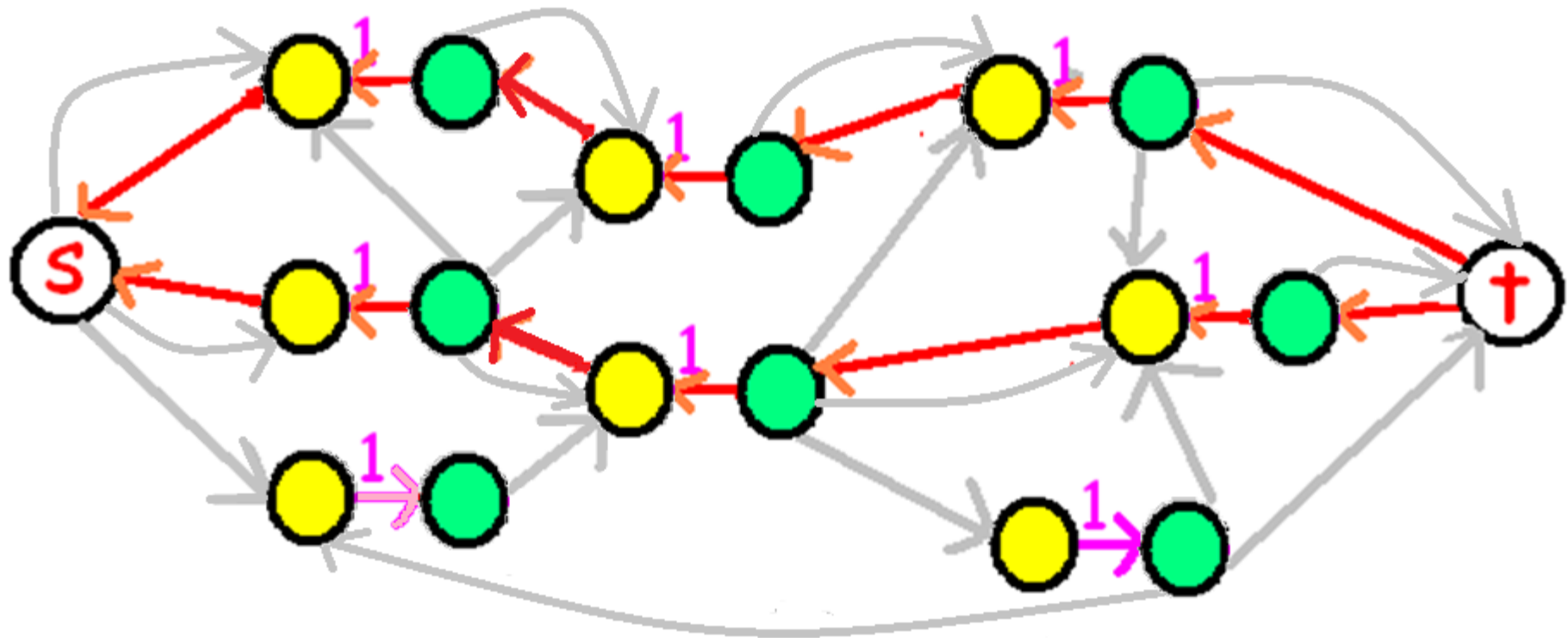
Pink arcs have capacity 1.

Other arcs have no limit (or you can use capacity 2)

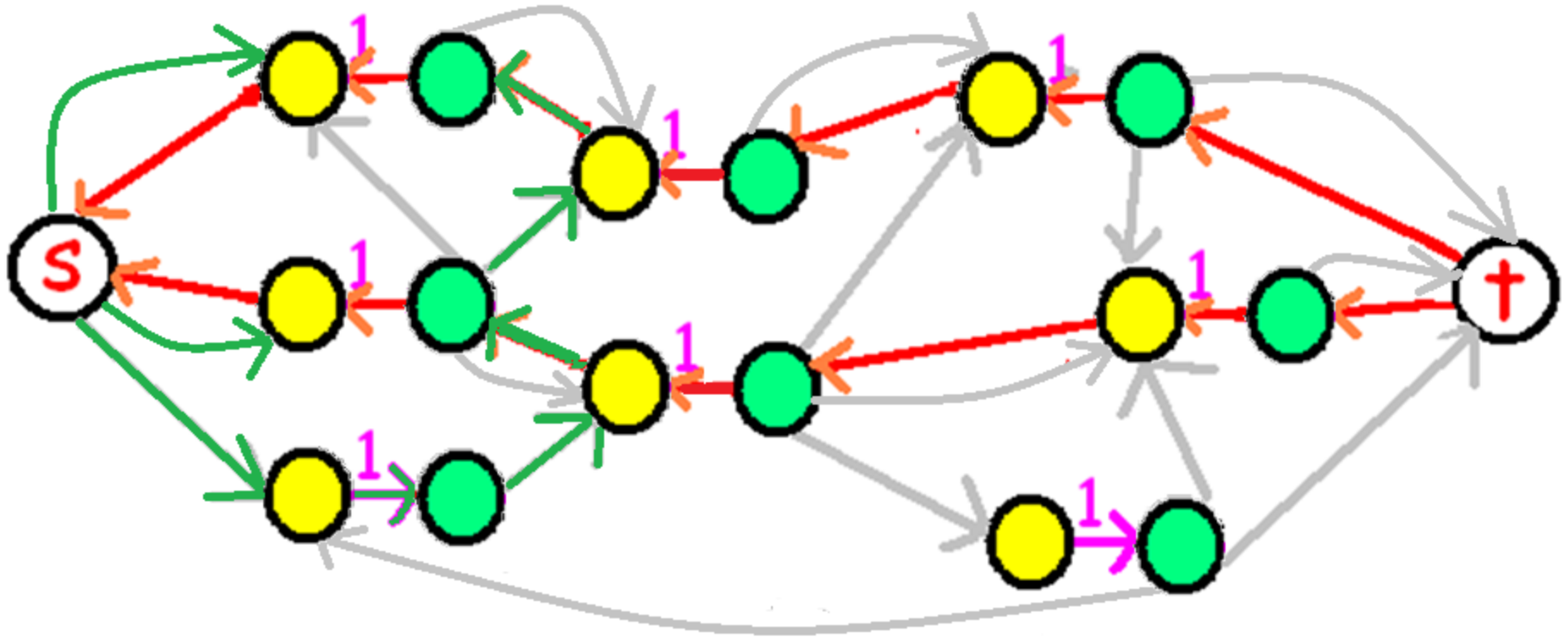




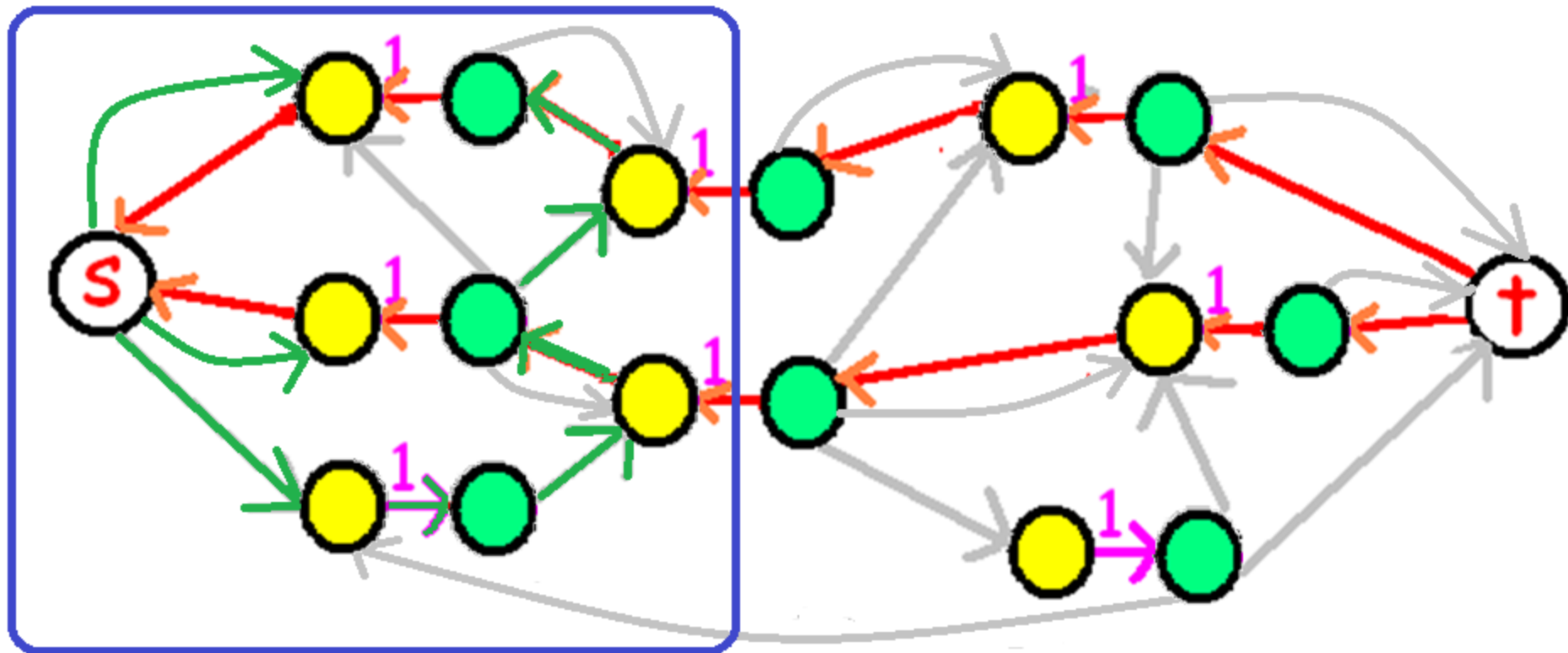
Maximum flow



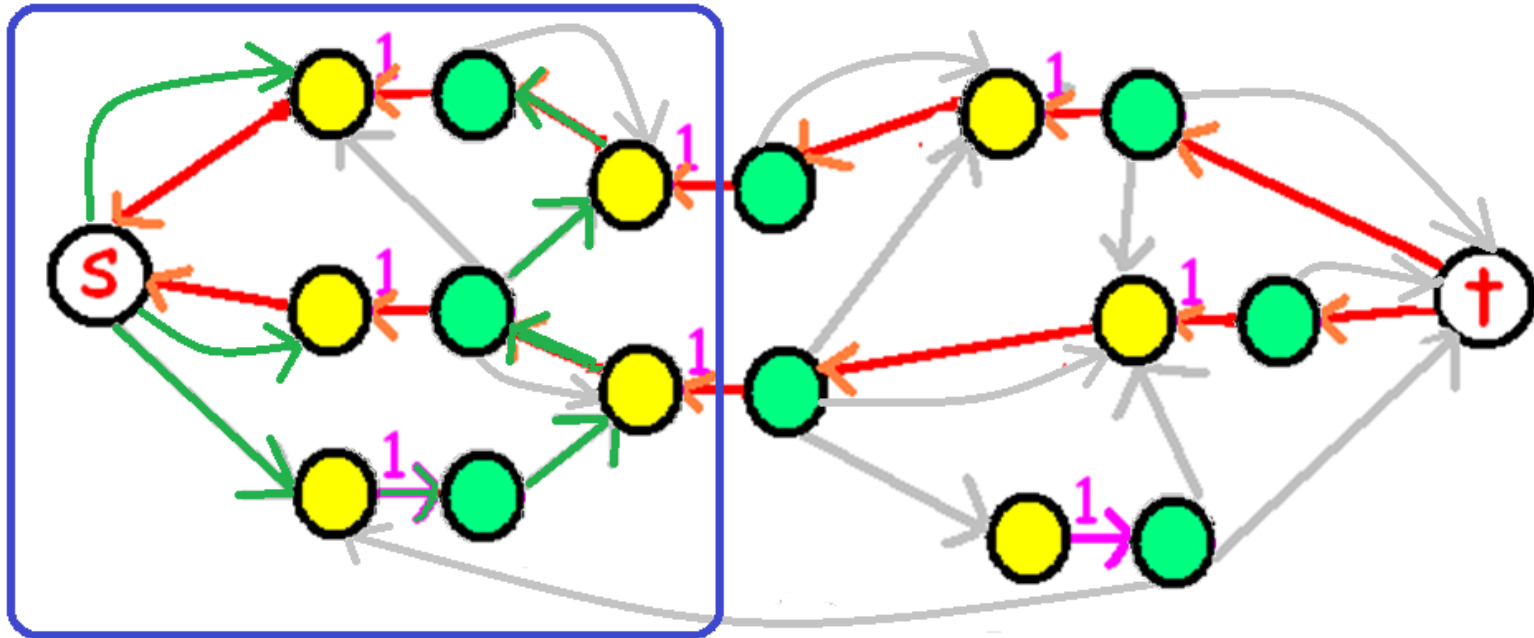
Auxillary graph.



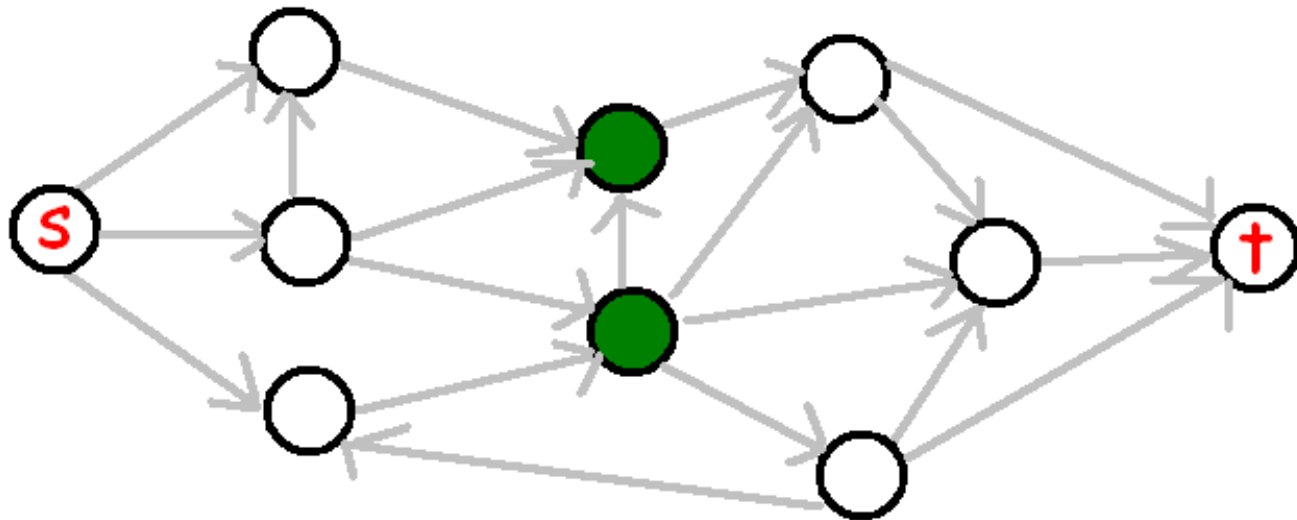
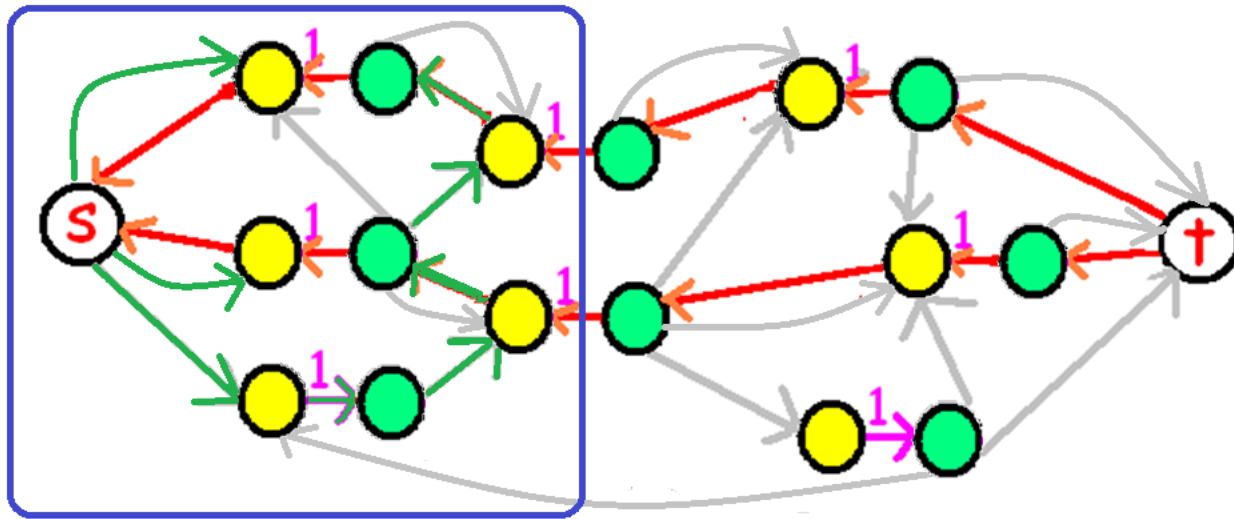
Green arcs are on BFS tree  
rooted at s.



The cut in this network.

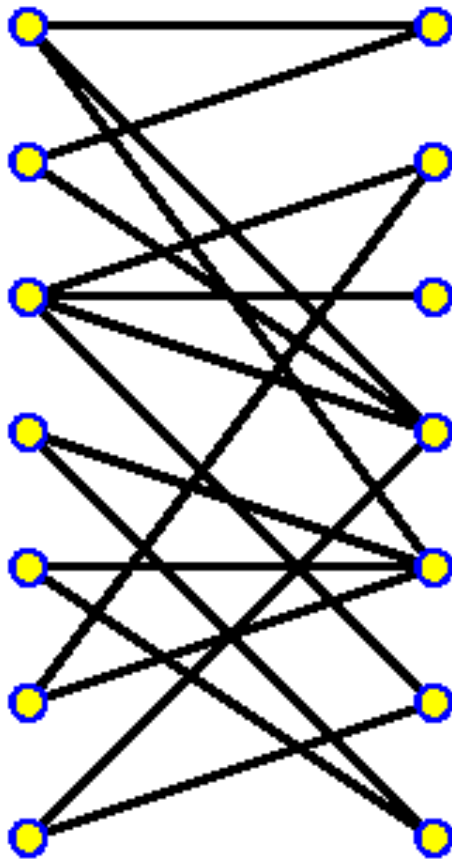


For each pink edge in the cut, its corresponding vertices are in the vertex cut of the original graph.

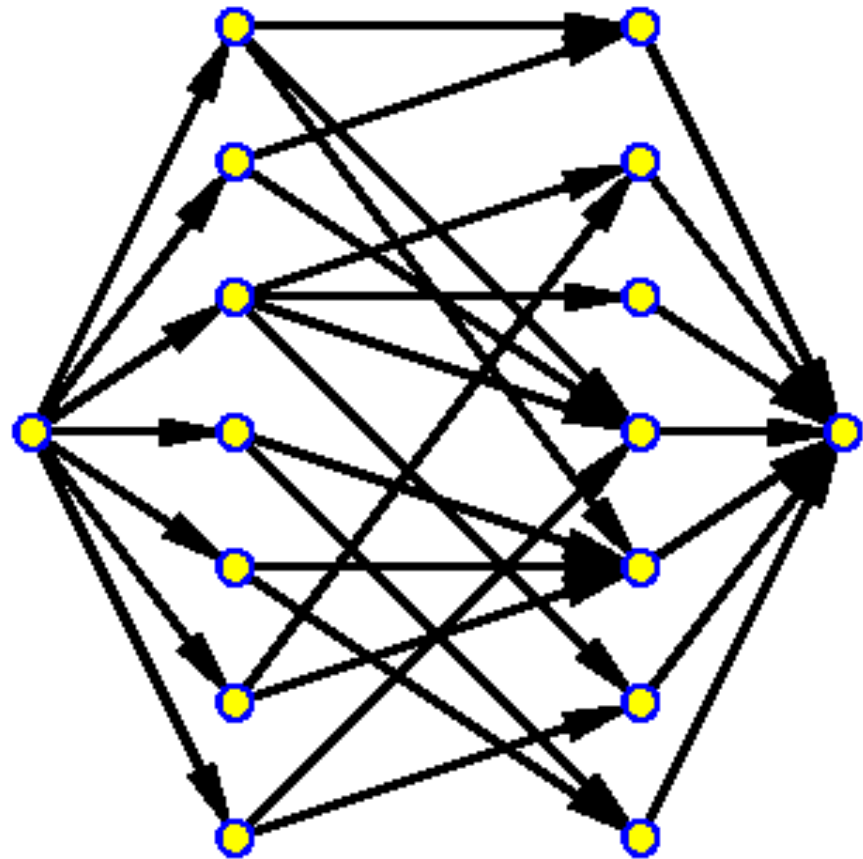


# USING MAXFLOW for BIPARTITE MATCHING

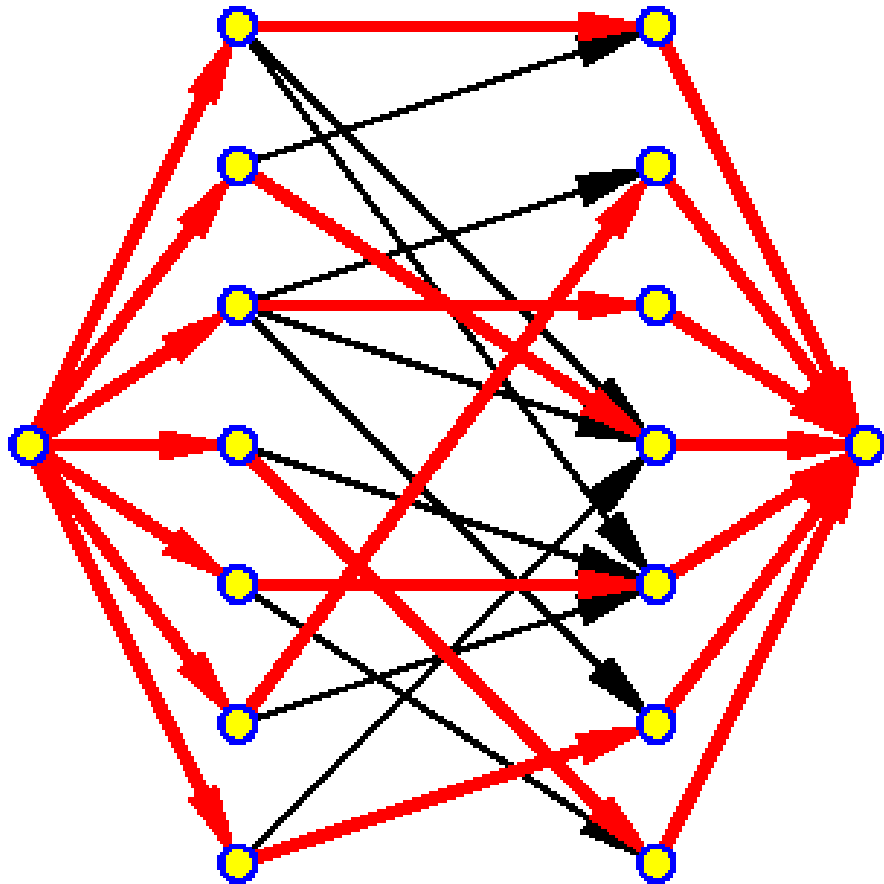
Bipartite Graph



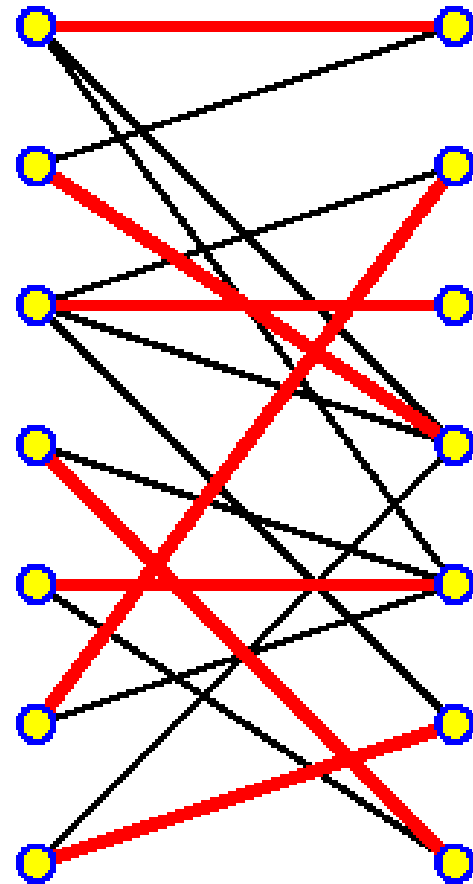
Corresponding Unit-Capacity Network



Maxflow in Network



Corresponding Matching



<http://www8.cs.umu.se/~jopsi/dinf504/chap14.shtml>





Ralph Gomery



T.C. Hu

The Gomory-Hu tree paper remains the most significant paper on multi-terminal flows since its publication in 1961.